Math 3710 HOMEWORK #8 Name:

Due 10/28/2022, 12:30, before start of the class.

Solve the following problems and staple your solutions to this cover sheet.

1. Sec 3.4 #6
2. Sec 3.5 #4
   Note: Are the answers in parts a and b the same? Hints: $1 \sim \sum_{n=1}^{\infty} \frac{2(1-(-1)^n)}{n\pi} \sin \frac{n\pi x}{L}$ and $x \sim \sum_{n=1}^{\infty} \frac{2(-1)^{n+1}L}{n\pi} \sin \frac{n\pi x}{L}$.

For problems 3-8, let $f(x) = x^2$, $h(x) = \frac{1}{3}x^3 - \frac{\pi^2}{3}x$ and $k(x) = x^3$ for $0 < x < \pi$. In an earlier homework we found the Fourier cosine series of f and by applying the convergence theorem, we have that $x^2 = \dfrac{\pi^2}{3} + \displaystyle\sum_{n=1}^{\infty} \frac{4(-1)^n}{n^2} \cos(nx)$ for $0 \leq x \leq \pi$. (Notice the equality holds at the endpoints. Do you know why?)

3. Show $x^2 = \displaystyle\sum_{n=1}^{\infty} \frac{2[(2-n^2\pi^2)(-1)^n - 2]}{n^3\pi} \sin(nx)$ for $0 \leq x < \pi$, by finding the Fourier sine series of f and discussing its convergence. Pay attention to the endpoints! Note: You may use Mathematica or Review, Identities, Theorems, Formulas and Tablesfor integration.
4. Determine which of the above two Fourier series of f can be differentiated term-by-term and use the appropriate one to find a Fourier series for $f'(x) = 2x$, $0 < x < \pi$. Explain!
   Hints: Determine whether f satisfies the required hypotheses for term-by-term differentiation of the Fourier cosine series or Fourier sine series. Then apply the term-by-term differentiation to the appropriate series.
5. Use the result of the last problem to show that $2x = \displaystyle\sum_{n=1}^{\infty} \frac{4(-1)^{n+1}}{n} \sin(nx)$ for $0 \leq x < \pi$ and $1 - \dfrac{1}{3} + \dfrac{1}{5} - \dfrac{1}{7} + \dfrac{1}{9} \cdots = \dfrac{\pi}{4}$.
   Hints: Solve the last problem correctly and apply the convergence theorem to the new Fourier series. Evaluate the equation at an appropriate x value. Note: Pay attention to the equality at $x = 0$.
6. Show $h(x) = \displaystyle\sum_{n=1}^{\infty} \frac{4(-1)^n}{n^3} \sin(nx)$ for $0 \leq x \leq \pi$, by term-by-term integration of the given Fourier cosine series of f.
   Notes: Notice that this is the Fourier sine series of h.
7. Use the results or problems 5 and 6 to find the Fourier sine series of $k(x)$.
   Hint: Fourier series operator is a linear operator.
8. Can we differentiate the Fourier sine series of $h(x)$ term-by-term? Explain! Can we differentiate the Fourier sine series of $k(x)$ term-by-term? What is the difference? Explain!
9. Free points!
10. Free points!