Math 3710 HOMEWORK #5 Name:

Due 9/30/2022, 12:30, before start of the class.

Solve the following problems and staple your solutions to this cover sheet.

1. Sec 2.3 #3(d) Notes: Show all steps in separation of variables. Either solve the EVP by discussing all three cases or **use the result of an earlier HW**. Either derive the formula for constants B_n or **use the result of an earlier HW**. Perform the integration to find the value of constants B_n. Since $u(x,\,0)$ is a step function, we must break up the integral from 0 to L to sum of integrals from 0 to $L/2$ and from $L/2$ to L. Also, $\cos n\pi = (-1)^n$.

2. Solve the following.

$$\begin{aligned} \frac{\partial u}{\partial t} &= \frac{\partial^2 u}{\partial x^2} &&,\ 0 < x < \pi\,,\ t > 0 \\ u(0,\,t) = u(\pi,\,t) &= 0 &&,\ t > 0 \\ u(x,\,0) &= x &&,\ 0 < x < \pi \end{aligned}$$

   Notes: State all steps. However, you may skip easy parts or recall work done in an earlier homework. Calculate the constants B_n. Yes, $k = 1$ and $L = \pi$ in this problem!

3. Find all nontrivial solutions of the eigenvalue value problem $\phi''(x) = -\lambda\,\phi$, for $0 \le x \le L$, and $\phi'(0) = \phi'(L) = 0$.
   Hints: Consider the cases $\lambda < 0$ or $\sqrt{-\lambda} = \mu > 0$, $\lambda = 0$ and $\lambda > 0$ or $\sqrt{\lambda} = \mu > 0$, in that order. For a peek at the solution, see Sec 2.4.1. The answer is also in Review, Identities, Theorems, Formulas and Tables handout.

4. Suppose $f(x) = A_0 + \displaystyle\sum_{n=1}^{\infty} A_n \cos\frac{n\pi x}{L}$ for $0 \le x \le L$. Show that $A_0 = \dfrac{1}{L}\displaystyle\int_0^L f(x)\,dx$ and $A_n = \dfrac{2}{L}\displaystyle\int_0^L f(x)\cos\frac{n\pi x}{L}\,dx$ for $n = 1,\,2,\cdots$. Hint: See class notes. You may interchange the order of integration and summation and use the orthogonality results from an earlier HW.

5. Solve the following.

$$\begin{aligned} \frac{\partial u}{\partial t} &= \frac{\partial^2 u}{\partial x^2} &&,\ 0 < x < \pi\,,\ t > 0 \\ \frac{\partial u}{\partial x}(0,\,t) = \frac{\partial u}{\partial x}(\pi,\,t) &= 0 &&,\ t > 0 \\ u(x,\,0) &= \cos x &&,\ 0 < x < \pi \end{aligned}$$

   Notes: State all steps. However, you may skip easy parts or recall the work done in an earlier homework or above. No integration is necessary for finding constants A_0 and A_n!

6. Find all nontrivial solutions of the boundary value problem $\phi''(x) = -\lambda\,\phi$, for $-L \le x \le L$, with periodic boundary conditions $\phi(-L) = \phi(L)$ and $\phi'(-L) = \phi'(L)$.
   Hints: Consider the cases $\lambda < 0$ or $\sqrt{-\lambda} = \mu > 0$, $\lambda = 0$ and $\lambda > 0$ or $\sqrt{\lambda} = \mu > 0$, in that order. There are two eigenfunctions for each positive eigenvalue! For a peek at the solution, see Sec 2.4.2. The answer is also in Review, Identities, Theorems, Formulas and Tables handout.

7. Suppose $f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{L} + \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{L}$ for $-L \le x \le L$. Show that $a_0 = \frac{1}{2L}\int_{-L}^{L} f(x)\,dx$, $a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\frac{n\pi x}{L}\,dx$ and $b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\frac{n\pi x}{L}\,dx$ for $n = 1, 2, \cdots$.
   Hint: See class notes. You may interchange the order of integration and summation and use the orthogonality results from an earlier HW.

8. Solve the following.

$$\begin{aligned}
\frac{\partial u}{\partial t} &= \frac{\partial^2 u}{\partial x^2} && , \ -L < x < L\,,\ t > 0 \\
u(-L,\, t) &= u(L,\, t) && , \ t > 0 \\
\frac{\partial u}{\partial x}(-L,\, t) &= \frac{\partial u}{\partial x}(L,\, t) && , \ t > 0 \\
u(x,\, 0) &= f(x) && , \ -L < x < L
\end{aligned}$$

   Notes: State all steps. However, you may skip easy parts or recall work done in an earlier homework or above.

9. Free points!

10. Free points!