Math 3710 HOMEWORK #4 Name:

Due 9/23/2022, 12:30, before start of the class.

Solve the following problems and staple your solutions to this cover sheet.

1. Sec 1.5 #9(a)
   Hints: Consider the heat equation in polar form. The temperature is circularly symmetric. Look for an equilibrium temperature of the form $u(r, \theta, t) = u(r)$. Also, it is easier not to expand $\frac{d}{dr}\left(r\,\frac{du}{dr}\right)$!
2. Sec 1.5 #10
   Hints: See hints above. In addition, since the origin is part of the domain, u should be defined at $r = 0$.
3. Sec 2.2 #2
   To prove an operator is not linear you just need to show it fails to meet the linearity condition in just one case (that meets the definition criteria).
4. Sec 2.2 #4
5. Consider the following equations.

   (i) $\frac{\partial^2 u}{\partial x^2} - 2\,\frac{\partial u}{\partial t} = 0$ (ii) $2u^2 + 3u + 1 = 0$ (iii) $(\frac{\partial u}{\partial x})^2 + \frac{\partial^4 u}{\partial y^4} = 0$

   (iv) $\frac{\partial^2 u}{\partial x^2} - u\,\frac{\partial u}{\partial t} = x$ (v) $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + u = 0$ (vi) $u''(x) + 2u'(x) = \sin x$

   (a) State which equations are partial differential equations along with their orders.
   (b) Determine which partial differential equations are homogeneous.
   (c) Determine which partial differential equations are linear.
   (d) Demonstrate why one of the partial differential equations you believe to be nonlinear is indeed not a linear equation.
   (e) Prove the linearity of one of the linear partial differential equations.
6. Find all nontrivial solutions of the eigenvalue value problem $\phi''(x) = -\lambda\,\phi$, for $0 \le x \le L$, and $\phi(0) = \phi(L) = 0$.
   Hints: Consider the cases $\lambda < 0$ or $\sqrt{-\lambda} = \mu > 0$, $\lambda = 0$ and $\lambda > 0$ or $\sqrt{\lambda} = \mu > 0$, in that order. See class notes. The answer is also in Review, Identities, Theorems, Formulas and Tables handout.
7. Suppose $f(x) = \sum_{n=1}^{\infty} B_n \sin\frac{n\pi x}{L}$ for $0 \le x \le L$. Show that $B_n = \frac{2}{L}\int_0^L f(x)\sin\frac{n\pi x}{L}\,dx$ for $n = 1, 2, \cdots$. Hint: See class notes. You may interchange the order of integration and summation and use the orthogonality results from an earlier HW.
8. Free points!
9. Free points!
10. Free points!