Math 3710 HOMEWORK #3 Name:

Due 9/16/2022, 12:30, before start of the class.

Solve the following problems and staple your solutions to this cover sheet.

1. Sec 1.2 #1 (a, b)
2. Sec 1.3 # 1
   Hint: Repeat the argument for equation 1.3.4. Also, see class notes.
3. Sec 1.4 # 1 (d, e)
   Hint: The PDE is $c\,\rho\,\frac{\partial u}{\partial t} = K_0\,\frac{\partial^2 u}{\partial x^2}$. The time independent solution satisfies $\frac{d^2 u}{dx^2} = -\frac{Q}{K_0}$.
4. Sec 1.4 # 1 (f, g)
   Hint: The PDE is $c\,\rho\,\frac{\partial u}{\partial t} = K_0\,\frac{\partial^2 u}{\partial x^2}$. The time independent solution satisfies $\frac{d^2 u}{dx^2} = -\frac{Q}{K_0}$.

In the next four problems, we derive the two-dimensional Laplacian in polar coordinates.

Note: In place of next four problems, you may solve problem #3 in section 1.5.

5. Show that $\dfrac{\partial r}{\partial x} = \cos\theta$, $\dfrac{\partial \theta}{\partial x} = -\dfrac{\sin\theta}{r}$, $\dfrac{\partial r}{\partial y} = \sin\theta$ and $\dfrac{\partial \theta}{\partial y} = \dfrac{\cos\theta}{r}$.
   Hint: See class notes.
6. Show that $\dfrac{\partial^2 u}{\partial x^2} = \cos^2\theta\,\dfrac{\partial^2 u}{\partial r^2} - \dfrac{2\sin\theta\cos\theta}{r}\,\dfrac{\partial^2 u}{\partial\theta\,\partial r} + \dfrac{\sin^2\theta}{r}\,\dfrac{\partial u}{\partial r} + \dfrac{\sin^2\theta}{r^2}\,\dfrac{\partial^2 u}{\partial\theta^2} + \dfrac{2\sin\theta\cos\theta}{r^2}\,\dfrac{\partial u}{\partial\theta}$.
   Hint: See class notes.
7. Show that $\dfrac{\partial^2 u}{\partial y^2} = \sin^2\theta\,\dfrac{\partial^2 u}{\partial r^2} + \dfrac{2\sin\theta\cos\theta}{r}\,\dfrac{\partial^2 u}{\partial r\,\partial\theta} + \dfrac{\cos^2\theta}{r}\,\dfrac{\partial u}{\partial r} + \dfrac{\cos^2\theta}{r^2}\,\dfrac{\partial^2 u}{\partial\theta^2} - \dfrac{2\sin\theta\cos\theta}{r^2}\,\dfrac{\partial u}{\partial\theta}$.
   Hint: See class notes.
8. Show that $\nabla^2 u = \dfrac{1}{r}\,\dfrac{\partial}{\partial r}\left(r\,\dfrac{\partial u}{\partial r}\right) + \dfrac{1}{r^2}\,\dfrac{\partial^2 u}{\partial\theta^2}$.
   Hint: See class notes.
9. Free points!
10. Free points!