Math 3710 HOMEWORK #2 Name:

Due 9/9/2022, 12:30 p.m., before start of the class.

Solve the following problems and staple your solutions to this cover sheet.

ODE Review: A general solution of the ODE $P(x)y'' + Q(x)y' + R(x)y = 0$ is of the form $y(x) = c_1 y_1(x) + c_2 y_2(x)$ where c_1 and c_2 are constants and 1. y_1 and y_2 are two solutions of this ODE and 2. y_1 and y_2 are linearly independent. Two solutions y_1 and y_2 of this ODE are linearly independent if their Wronskian is not zero: $W(y_1, y_2) = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = y_1 y_2' - y_2 y_1' \neq 0$.

For the next three problems, consider the ODE $\phi''(x) = r^2\,\phi(x)$, where r is a positive constant. We know that $\phi_1(x) = e^{-rx}$ and $\phi_2(x) = e^{rx}$ are two linearly independent solutions of it and its general solution is $\phi(x) = c_1\,e^{-rx} + c_2\,e^{rx}$.

1. Show that $\phi(x) = c_1 \cosh(rx) + c_2 \sinh(rx)$ is also a general solution of the above ODE.
2. Show that $\phi(x) = c_1 \cosh[r(x - x_0)] + c_2 \sinh[r(x - x_0)]$, where x_0 is a constant, is another general solution of the above ODE.
3. Show that $\phi(x) = c_1 \sinh[r(x - x_0)] + c_2 \sinh(rx)$, where x_0 is a **nonzero** constant, is yet another general solution of the above ODE. Note: $\sinh z = 0$ if and only if $z = 0$.

Hints for Problems 1-3: In each case, show that 1. each of the given functions is a solution of the ODE and 2. the two given functions are linearly independent. For the definitions and properties of hyperbolic trigonometric functions, $\sinh x$ (pronounced "cinch of x") and $\cosh x$ (pronounced "kosh of x"), see Review, Identities, Theorems, Formulas and Tables.

4. Suppose f is an even differentiable function. Show that f' is an odd function.
   Hint: Since f is an even function, $f(-x) = f(x)$ for all x in the domain of f. Differentiate $f(-x) = f(x)$ using implicit differentiation.
5. Suppose f is an odd differentiable function. Show that f' is an even function.
   Hint: Since f is an odd function, $f(-x) = -f(x)$ for all x in the domain of f. Differentiate $f(-x) = -f(x)$ using implicit differentiation.
6. Suppose differentiable function f is periodic with period p. Show that f' is also a periodic function with period p.
   Hint: Since f is p-periodic, $f(x + p) = f(x)$ for all x in the domain of f. Differentiate $f(x + p) = f(x)$ using implicit differentiation.
7. Suppose (piecewise) continuous function g is periodic with period p. Show that
   $\displaystyle\int_c^{c+p} g(x)\,dx = \int_0^p g(x)\,dx$ for any number c.
   Hints: $\int_c^{c+p} g(x)\,dx = \int_c^p g(x)\,dx + \int_p^{c+p} g(x)\,dx$. In the second integral, apply the u-substitution, $u = x - p$, and the periodicity property of g. Then, combine the two integrals.

8. Suppose (piecewise) continuous function g is periodic with period p and $\displaystyle\int_0^p g(x)\,dx = 0$. Show that $G(x) = \displaystyle\int_0^x g(t)\,dt$ is also a periodic function with period p.
   Hint: Write $G(x+p) = \int_0^{x+p} g(t)\,dt$ as the sum of two appropriate integrals and use the result of the last problem to show $G(x+p) = G(x)$.

9. Free points!

10. Free points!