Math 3710 HOMEWORK #11 Name:

Due 12/9/2022, 12:30, before start of the class.

Solve the following problems and staple your solutions to this cover sheet. You may use results in Review, Identities, Formulas and Theorems handout.

1. Sec 7.7 #1
   Hints: Assume $u(r,\,\theta,\,t) = f(r)\sin 3\theta\, h(t)$ and that u is defined at $r = 0$. You may use results in Review, Identities, Formulas and Theorems handout.

2. Sec 7.7 #10
   Hints: Assume $u(r,\,\theta,\,t) = f(r)g(t)$ and that u is defined at $r = 0$. You may use results in Review, Identities, Formulas and Theorems handout.

3. Sec 7.8 #2(a, b)
   Hint: You can show $\lambda > 0$ by either discussing the three cases, $\lambda < 0$, $\lambda = 0$, and $\lambda > 0$ or deriving a Raleigh type quotient for λ.

4. Sec 7.9 #1(b)
   Hints: Assume $u(r,\,\theta,\,z) = f(r)\sin 7\theta\, h(z)$ and that u is defined at $r = 0$. You may use results in Review, Identities, Formulas and Theorems handout.

5. Sec 8.2 #1(b)
   Hints: There is no equilibrium solution! In addition to showing this mathematically, try to give a physical reason for it. Find a reference function $r(x)$ such that $r'(0) = 0$ and $r'(L) = B$. You are asked to only reduce the problem to a one with homogeneous boundary conditions.

6. Sec 8.2 #2(b)
   Hints: Look for reference function $r(x,\,t)$ such that $r(0,\,t) = A(t)$ and $\frac{\partial r}{\partial x}(L,\,t) = B(t)$. You are asked to only reduce the problem to a one with homogeneous boundary conditions.

7. Solve

$$
\begin{aligned}
\frac{\partial u}{\partial t} &= k\,\frac{\partial^2 u}{\partial x^2} - \frac{x}{L}, && 0 < x < L\,,\ t > 0\\
u(0,\,t) &= 0, && t > 0\\
u(L,\,t) &= 0, && t > 0\\
u(x,\,0) &= 0, && 0 < x < L
\end{aligned}
$$

   Hint: $u_E = \frac{1}{6kL}\,x^3 - \frac{L}{6k}\,x$. Find the constants. You may use Mathematica and results of an earlier problem!

8. Sec 8.2 #6(a)

9. Sec 8.2 #6(b)
   Hint: $u_E = -\frac{1}{2c^2}\,x^2 + \frac{L}{2c^2}\,x$.

10. Free points!