Math 3550 HOMEWORK #3 Name:

Due 9/18/2023, 11:30 a.m.

Solve the following problems and staple your solutions to this cover sheet. (Computer outputs must be put in the appropriate place in the solution, not attached as an appendix. You may physically cut and paste the output in the problem or allow appropriate space in the printout to add your hand written work.)

1. Sec 1.3, Prob 1(d).
2. Sec 1.3, Prob 2(h) and 3(b). See the Mathematica notebook for this section, your first HW, or the Mathematica commands below.
3. Sec 1.3, Prob 8. Similar to problem 7 in this section. Set $p = 1500$ and experimentally find the largest a_0 value for which $a_{360} = 0$.
4. Sec 1.4, Prob 4. See the Mathematica notebook for this section or the Mathematica commands below.
5. Sec 1.4, Prob 7. See the Mathematica notebook for this section or the Mathematica commands below.
6. Sec 2.2, Prob 12.
7. Sec 2.2, Prob 13. See example 1.
8. Sec 2.2, Prob 14. Discuss your ideas in details.

Mathematica Commands

The numerical values of a recurrence relation $a_{n+1} = f(a_n)$ with initial condition a_0 for $n = 0$ to $n = m$ can be obtained the following way.

```
RecurrenceTable[{a[n+1]==f(a_n), a[0]==a_0}, a, {n, 0, m}]
```

To plot a simple recurrence relation $a_{n+1} = f(a_n)$ with initial condition a_0 for $n = 0$ to $n = m$ do the following.

```
a[0]=a_0;
a[n_]:=f(a_{n-1})
ListPlot[Table[{n, a[n]},{n, 0, m}]]
```

To plot anything but a very simple recurrence relation $a_{n+1} = f(a_n)$ with initial condition a_0 for $n = 0$ to $n = m$ do the following.

```
xvalues = Table[n, {n, 0, m}];
yvalues = RecurrenceTable[{a[n] == f(a_{n-1}), a[0] == a_0}, a, {n, 0, m}]
points = Transpose[{xvalues, yvalues}]
ListPlot[points]
```

To plot a simple recurrence relation $a_{n+1} = f(a_n)$ with several initial condition values a_{01}, a_{02}, ..., a_{0k} for $n = 0$ to $n = m$ on the same coordinate system, do the following.

```
a[n_]:=f(a_{n-1})
ListPlot[Table[Table[{n, a[n]},{n, 0, m}],{a[0],{a_01, a_02, ...  , a_0k}}]]
```

To plot a very simple system of recurrence relations $a_{n+1} = f(a_n, b_n)$, $b_{n+1} = g(a_n, b_n)$ with initial condition a_0 and b_0 for $n = 0$ to $n = m$, as ordered pairs (a_n, b_n), do the following. Otherwise, skip to the next parts.

```
a[0]=a_0; b[0]=b_0;
a[n_]:=f(a_{n-1}, b_{n-1}); b[n_]:=g(a_{n-1}, b_{n-1})
ListPlot[Table[{a[n], b[n]},{n, 0, m}]]
```

The numerical values of a system of difference equations $a_{n+1} = f(a_n, b_n)$, $b_{n+1} = g(a_n, b_n)$ with initial condition a_0 and b_0 for $n = 0$ to $n = m$ can be obtained the following way.

```
RecurrenceTable[{a[n+1]==f(a_n, b_n), b[n+1]==g(a_n, b_n), a[0]==a_0, b[0]==b_0},
                                                            {a, b}, {n, 0, m}]
```

To plot these values as an ordered pair (a_n, b_n) do the following way.

```
model=RecurrenceTable[{a[n+1]==f(a_n, b_n), b[n+1]==g(a_n, b_n), a[0]==a_0, b[0]==b_0},
                                                            {a, b}, {n, 0, m}]
ListPlot[model, PlotRange->All]
```

To plot the values of each variable at each time period for the system of difference equations $a_{n+1} = f(a_n, b_n)$, $b_{n+1} = g(a_n, b_n)$ with initial conditions a_0 and b_0 for $n = 0$ to $n = m$ on the same coordinate system do the following.

```
model=RecurrenceTable[{a[n+1]==f(a_n, b_n), b[n+1]==g(b_n, b_n), a[0]==a_0, b[0]==b_0},
                                                            {a, b}, {n, 0, m}]
amodel=Table[{i-1, model[[i, 1]]}, {i, 1, m + 1}]
bmodel=Table[{i-1, model[[i, 2]]}, {i, 1, m + 1}]
ListPlot[{amodel, bmodel},
 PlotStyle->{{Red, PointSize[0.02]}, {Blue, PointSize[0.01]}}, PlotRange->All]
```

The a values will be plotted in larger red color dots.

It is possible to plot values of a_n, b_n, or both, versus time period n or plot ordered pairs (a, b) at each time period while varying any of the data, like initial values a_0, b_0, or functions f or g, on the same coordinate system by forming tables. However, it might be difficult to distinguish between them.