MATH 3420 HOMEWORK #4 Name:

Due Tuesday, Mar 5

Solve the following problems and staple your solutions to this cover sheet.

1. Exercise 5.90

2. Exercise 5.91

3. Exercise 6.2

4. Exercise 6.5

5. Exercise 6.7

6. Exercise 6.11

7. Exercise 6.16

8. Let Y be a random variable with probability density function

$$f(y) = \begin{cases} 6y(1-y) & \text{for } 0 \le y \le 1 \\ 0 & \text{elsewhere} \end{cases} .$$

Find the probability density function for the random variable $U = 3Y - 2$.

9. Let Y_1 and Y_2 be continuous random variables with joint probability density function

$$f(y_1, y_2) = \begin{cases} \frac{1}{8}(6 - y_1 - y_2) & , 0 < y_1 < 2 \text{ and } 2 < y_2 < 4 \\ 0 & , \text{otherwise.} \end{cases}$$

Find the probability density function for the random variable $U = Y_1 + Y_2 + 2$.

Hint: Consider $F(u) = P(U \le u)$ for the cases $u < 2$, $2 \le u < 6$ $6 \le u < 8$, and $u \ge 8$.

10. Suppose that Z is a normally distributed random variable with mean $\mu = 0$ and variance $\sigma^2 = 1$. Show that Z^2 has χ^2 distribution with $\nu = 1$ degree of freedom.

Hints: Let $f(z)$ be the standard normal density function and set $U = Z^2$. $F(u) = 0$, if $u < 0$. $F(u) = 2\int_0^{\sqrt{u}} f(z)\,dz$, if $u \ge 0$. Then use the Fundamental Theorem of Calculus and the fact that $\Gamma(\frac{1}{2}) = \sqrt{\pi}$.