Probability and Statistics
Sample Exam II Questions

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4 pages

Sections 3.6 - 4.12 & 7.1-7.2
(except sections 3.10 & 4.11)
Math 3410, ID #1967501, Fall 2001

1. What is the probability that the first six is observed in the fourth roll of a fair die? What is the probability that the second six is observed in the seventh roll? (The first six can occur at any of the earlier rolls.)

2. Suppose that a basketball player can make a free throw 60% of the time. Let Y equal the minimum number of free throws that this player must attempt to make a total of 10 shots. Find the mean and variance of Y.

3. A clearance sales bin contains two types of toys; 20 dolls and 15 cars. Suppose a child chooses three toys in random from the 35 toys in the bin. What is the probability that all three toys are cars? If a doll costs \$10 and a car costs \$17, what is the expected value of the total cost?

4. A school district plans to purchase 30 new computers from a local store which has 200 computers in stock in which 10 are defective. Assuming computers are chosen in random from the stock, what is the probability of purchasing 3 defective computers?

5. Customers arrive at a checkout counter in a department store according to a Poisson distribution at a average of ten per hour.
   (a) For a given one hour period, find the probability that exactly six customers arrive.
   (b) For a given one hour period, find the probability that more than three customers arrive.
   (c) For a given two hour period, find the probability that exactly 15 customers arrive.

6. The number of defects Y for a particular brand of new cars is assumed to have a Poisson distribution with mean 2. What is the probability that a car chosen in random has 3 defects? If a husband and wife each buy one of these cars, what is the probability that the total number of defects on their two new cars is 3? Assume they choose the cars in random.

7. The number of accidents at an intersection follows a Poisson distribution. During the month of April, which has 30 days, one accident was reported. Find the probability that the accident occurred during the last half of the month. Find the probability that the accident occurred on April 16th. If no accident was reported in the first 15 days, what is the probability that the accident occurred on April 16th?

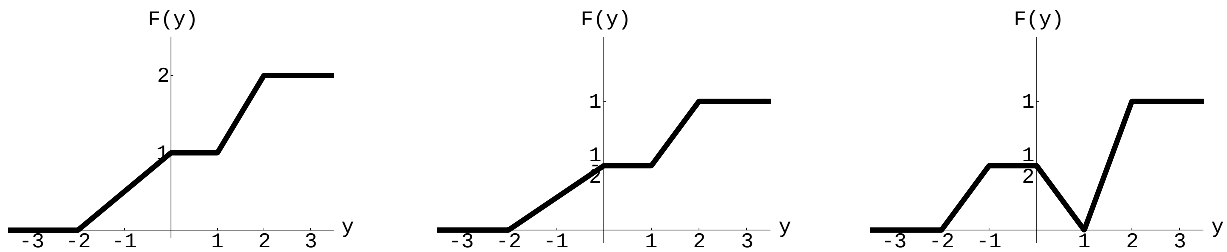
8. Suppose Y is a Poisson random variable with parameter λ. Find its moment-generating function.

9. The moment-generating function of the random variable Y is $m(t) = \frac{1}{6}e^{2t} + \frac{1}{3}e^{-t} + \frac{1}{2}e^{5t}$. Find the mean and variance of Y. Find the distribution of Y and justify your answer.

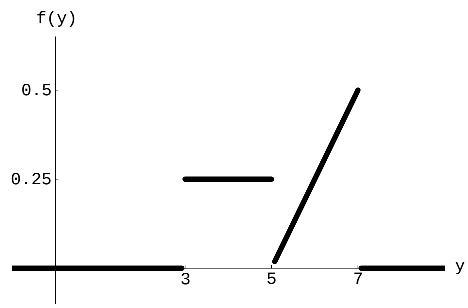
10. Let Y be a random variable with mean 20 and variance 5. Find b such that $P(|Y - 20| > b) \leq 0.225$.

11. Let Y be a random variable with moment-generating function $m_Y(t)$. Consider the random variable $U = aY + b$, where a and b are real-valued constants. Show that the moment-generating function of U is $m_U(t) = e^{tb} m_Y(at)$.

12. One of the following is a graph of a (cumulative) distribution function. Identify it and verify that it satisfies the properties of distribution functions. Explain why the other two are not graphs of distribution functions. Then, draw the graph of the probability density function corresponding to the distribution function whose graph you have selected.



13. The probability density function of the continuous random variable Y is graphed below.



Find the following.

(a) $P(3.5 < Y < 5)$

(b) $P(4 < Y < 6)$

(c) Find the number b such that the maximum value of $P(y < Y < y+1)$ over all possible values of y is $P(b < Y < b+1)$. You may justify your answer graphically.

14. Let Y be a random variable with probability density function

$$f(y) = \begin{cases} ay^2(4-y) & , 0 \le y \le 4 \\ 0 & , \text{otherwise} \end{cases}$$

where a is an appropriate constant. Find $E(Y)$, $V(Y)$ and $P(Y \ge 3)$.

15. Let Y be a random variable with probability density function

$$f(y) = \begin{cases} ay + b & , 0 \le y \le 1 \\ 0 & , \text{elsewhere} \end{cases}$$

where a and b are appropriate constants. Find a and b so that $P(Y < \frac{1}{2}) = \frac{3}{8}$.

16. Let Y be a random variable with pdf

$$f(y) = \begin{cases} c(1-y^2) & \text{for } 0 \le y \le 1 \\ 0 & \text{elsewhere} \end{cases}$$

where c is an appropriate constant. Find $F(Y)$ and use it to calculate $P(Y > \frac{1}{2})$.

17. The cycle time for trucks hauling concrete to a highway construction site in uniformly distributed over the interval 45 to 75 minutes. What is the probability that the cycle time is at most 60 minutes? What is the probability that the cycle time exceeds 70 minutes if it is known that the cycle time exceeds 60 minutes?

18. Find the exact value of the following integral.
$$\int_0^\infty \frac{y^7 e^{-\frac{y}{2}}}{2^4\Gamma(4)}\,dy$$

19. The number of arrivals Y at the registrars counter in a one hour period has a Poisson distribution with mean 5. Let X denote the length of the time until the first arrival in a given one hour period. Find the cumulative distribution function of X and show that X is exponentially distributed.

20. Suppose the life span of a particular electronic equipment is exponentially distributed with a mean of 12,500 hours. What is the probability that one chosen in random will last 10,000 hours? What is the probability that one chosen in random will last 20,000 hours given that it has been operational for 10,000 hours?

21. The annual rainfall, in inches, in a certain region is normally distributed with mean 40 and variance 36. What is the probability that the annual rainfall will be between 34 and 52 inches? What is the probability that the annual rainfall will exceed 47.5 inches? Assuming that the annual rainfall in consecutive years are independent random variables, what is the probability that the rainfall will exceed 47.5 inches for two years in a row?

22. Suppose Z is a standard normal random variable.
    (a) Find z_1 such that $P(Z < z_1) = 0.8729$
    (b) Find z_2 such that $P(Z > z_2) = 0.2965$

23. The random variable Y has a normal distribution with mean 10 and variance 4 and X is a random variable having a χ^2 distribution with 5 degrees of freedom. Find the following.
    (a) $P(6.5 < Y < 14.26)$
    (b) y such that $P(Y > y) = 0.2345$
    (c) x such that $P(X < x) = 0.025$

24. Let Y be an exponential random variable with mean $\mu = \beta$ and variance $\sigma^2 = \beta^2$. Find $P(|Y - \mu| \leq 2\sigma)$.

25. Let Y be an exponential random variable with mean 5. Consider the random variable $X = Y^2$. Find the mean and variance of X.

26. Let Y be an exponential random variable with parameter β. Prove that $E(Y) = \beta$ and $V(Y) = \beta^2$.

27. According to a recent report Americans spend an average of \$1,400 per person per year on recreational activities with the standard deviation of \$700. Assume that observations are normally distributed. Find the probability that a person chosen at random spends between \$1,200 and \$1,600 per year.

28. Suppose $Z_1, \cdots, Z_5$ is a random sample from the standard normal distribution, find a number z such that $P\left(\sum_{i=1}^{5} Z_i^2 \leq z\right) = 0.95$.

29. Consider a normally distributed population with mean 12 and variance 4. Suppose n sample points $Y_1, \cdots, Y_n$ are chosen randomly from this population. Let $\overline{Y} = \sum_{i=1}^{n} Y_i$. Find the value of n for which $P(|\overline{Y} - 12| < 0.5) = 0.95$.

30. Suppose a population is normally distributed with mean μ and standard deviation $\sigma = 1.00$.

    (a) How many observations should be included in a sample if we wish the sample mean $\overline{Y}$ to be within 0.307 of μ with probability 0.95.

    (b) For the sample size obtained in (a), find the numbers c and d such that $P(c \le S^2 \le d) = 0.95$ where S^2 is the variance of the sample.

31. Let $Z_1, \cdots, Z_5$ be an independent random sample from a standard normal distribution and let $Y = \sum_{i=1}^{5} Z_i^2$. Suppose the random variable X, which is independent from the Z's and Y, has a χ^2 distribution with 4 degrees of freedom. Find a number a such that $P(Y \le a) = 0.95$. Find a number b such that $P\left(\frac{Z_1}{\sqrt{\frac{X}{4}}} > b\right) = 0.90$. Find a number c such that $P\left(\frac{\frac{Y}{5}}{\frac{X}{4}} > c\right) = 0.025$.