Probability and Statistics — A. Ghoreishi

Sample Exam I Questions — 3 pages

Sec 1.1 - Sec 3.5

Math 3410, ID #1967501, Fall 2001

1. Find the sample mean, variance and standard deviation of the sample space $\{1, 2, 3, 4, 10\}$.
2. Construct a relative frequency histogram for the following data. Use at least 4 subintervals and show the details of your work.

$$\{87, 109, 79, 80, 96, 95, 90, 96, 98, 101, 91, 78, 112, 94, 98, 94, 107, 81, 96\}$$

3. Find the sample mean and the sample variance of the following sample space. Draw a relative frequency histogram of this data using 5 subintervals.

$$S = \{1, 4, 7, 10, 9, 8, 7, 5, 12, 11, 15, 13, 6, 5, 7\}$$

4. The amount of water put in bottles is normally distributed with a mean of ten ounces and a standard deviation of one ounce. About what percent of the bottles will contain between nine and eleven ounces of water?
5. How many different seven-digit telephone numbers can be formed if the first digit cannot be zero, the second digit is odd and the whole number is divisible by 5?
6. How many different signals can be made using four flags of different colors on a vertical flagpole if exactly three flags are used for each signal?
7. Suppose 13 people are invited for dinner, of which 4 are men, 3 are women and the rest are children. If members of each group are seated next to each other, how many different seating arrangements are possible? Assume all are seated in one row.
8. How many poker hands (5 cards) are possible using a standard deck (52 cards)? How many different flush hands (all five the same suite, there are 4 different suites, each consisting of 13 different cards) are possible?
9. A company has three openings.
   (a) If 5 people have applied for the first position, 3 people have applied for the second and 8 people for the 3rd, how many ways can the company fill its openings? Assume all candidates for each job are equally qualified.
   (b) Suppose the three positions are distinct. If a total of 8 people, which qualify for all three positions, have applied, in how many ways can the company fill its vacancies?
   (c) Suppose the company plans to hire at least one female. If a total of 8 males and 6 females have applied, in how many ways can the company fill its open positions? Assume the three jobs are equivalent and all applicants are equally qualified.
10. A fair coin is tossed four times, and the sequence of the heads and tails is observed. List all elements of the sample space. Let events A, B, C and D be given by $A = \{\text{at least 3 heads}\}$, $B = \{\text{at least 2 heads}\}$, $C = \{\text{heads on the third toss}\}$, and $D = \{\text{1 head and 3 tails}\}$. Find $P(A)$, $P(A \cap B)$, $P(B)$, $P(A \cap C)$, $P(D)$, $P(A \cup C)$, and $P(B \cap D)$.
11. If five people are randomly seated in a row of five seats, what is the probability that a particular two of them are not sitting next to each other?

12. What is a probability of a full house (a three of a kind along with a two of a kind) poker hand if the five cards are chosen randomly from a standard 52 card deck (13 different groups, each consisting of four cards of the same kind)?

13. What is the probability of getting 4 of a kind if 5 cards are chosen in random from a standard 52 deck of card (13 different groups of 4 of a kinds)?

14. Eight objects are distributed randomly among three barrels with three in the first barrel, two in the second barrel, and three in the third. What is the probability that a particular one of the objects is in the first barrel?

15. Twelve balls numbered 1 through 12 are to be randomly placed in 3 buckets so that bucket A will contain three balls, bucket B will contain five balls and the rest will be placed in bucket C. In how many ways can this be done? Suppose exactly three of the balls are red in color. What is the probability that all three red balls will be placed in the Bucket C? What is the probability that one red ball will be placed in each bucket?

16. In a manufacturing process, production line A produces 30% of the output and 4% of the items it produces are defective. Line B produces 50% of the output with a defective rate of 8%, and line C produces the remaining and has 3% defective rate. Suppose outputs of each line are boxed in lots of 10, and any large shipment consists of outputs of all lines in proportion of production. If one out of three items sampled by the customer from a box in a large shipment are defective, what is the probability that box came from line B?

17. An item is produced in only two different locations. Location A produces 70% of the items and has a defective rate of 6% and 4% of the items produced in location B are defective. Suppose items from each location are boxed separately in lots of five and orders are filled in proportion of production from the two locations. If one out of three items sampled by a customer from a box in a large shipment are defective, what is the probability that the box came from production location A?

18. Suppose that $P(A) = 0.3$, $P(B) = 0.6$ and $P(A \cup B) = 0.7$. Find $P(A \cap B)$ and $P(B|A)$.

19. Consider events A, B and C in the sample space S. Suppose events B and C are disjoint and $B \cup C = S$. If $P(B) = 0.6$, $P(A|B) = 0.2$ and $P(A \cap C) = 0.38$, find $P(A \cap B)$, $P(A)$ and $P(B|A)$.

20. A drawer contains four black, six brown and eight olive socks. Two socks are selected at random. What is the probability that both socks are the same color? What is the probability that both socks are olive if it is known that they are the same color?

21. Events A and B are independent. Show that the events A and $\overline{B}$ are also independent.

22. A single die is cast. If one or two turns up, the random variable Y is the number showing. If the number is different from one or two, then two fair coins are flipped and random variable Y is the number of the heads observed. Find the probability function of Y.

23. In a coin game, four fair coins are flipped simultaneously. If only one head is observed, the pay off is \$1. If two or more heads are observed, the pay off is \$5. If four tails is observed, the pay off is \$10. Define the random variable Y to be the game's pay off. Find its probability function $P(Y = y)$, its expected value $E(Y)$, and its variance $V(Y)$.

24. If two fair coins are flipped and the random variable Y is the number of heads observed, Find $E(Y^2)$ and $V(Y)$.

25. Let Y be a discrete random variable. Prove that

$$E(aY+b) = aE(Y)+b \text{ and } V(aY+b) = a^2\,V(y)$$

where a and b are constants.

26. Suppose it is known that 10% of new cars require a visit to the dealer after the purchase to fix one or more faults. If a dealer sells five new cars, what is the probability that at least two of the cars will require a return visit for a fix up?

27. If the survival rate for a particular operation is only 60%, what is the probability that at least two out of the next five patients survive?

28. Suppose the probability that a lottery ticket is a winner is p. How many tickets should you buy if you want the probability of having at least one winning ticket to be no less than x?

29. In a binomial distribution, the mean is 4 and the variance is $\frac{8}{3}$. Find p.

30. What is the probability that the first six is observed in the fourth roll of a fair die?

31. Apples are packaged automatically in 3-pound bags. Suppose that 4% of the time the bag of apples weigh less than 3 pounds. If you select bags randomly and weigh them in order to find one underweight bag of apples, find the probability that the number of bags that must be selected is exactly 20, at least 20, or at most 20.