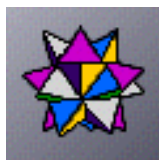


# An Introduction to Mathematica

## Math 3410




1. The Mathematics computer lab is located in Bldg 4, Rm 505. All computers in this lab have Mathematica 4.0 installed. Other PC computer labs should also have this software.

2. Here is how to get on Mathematica:

Get on a computer with the Mathematica software installed. (Some machines may require a log in process. Instruction and help should be available in the labs.) Move the mouse pointer to the **Mathematica icon**, which looks like the above star like symbol, and rapidly **click twice** using the **left mouse button**.

3. Entering information is like using a word processor. You can also use the arrow and backspace keys for editing. Try typing some text and practice editing it. Mathematica displays all work in “cells”. The brackets on the far right indicate the cell. You can create a new cell for your next input by clicking the mouse below the cell you have been working in. First a horizontal line will appear and then when you type a new cell will be created.

**Throughout this lab input the statements in True Type font exactly as is!**

4. Mathematica will do arithmetic operations using the usual symbols  $+$ ,  $-$ ,  $*$ ,  $/$ ,  $^$ . A blank space between two terms also indicates multiplication but using  $*$  might facilitate later editing. To see how it works, enter the following calculation followed by the appropriate **Evaluation** (Execution) key command; the **Enter** button on the number pad (not  on the main keyboard) should work on all computers.

4\*8  (in the number pad, not the Return key)

What you typed will now be given a label of *In[1]:=* (for input line 1). The answer will be in the next line labeled *Out[1]:=* (output line 1). Now use Mathematica to evaluate the following. Remember to push the **Enter** key after each part. Check the answers using a calculator. If the output is the same as the input, you probably have made an error in typing.

- (a)  $3+6*5$  (Does Mathematica follow order of operation rules?)
- (b)  $274/3$  (Try this again with a decimal point after either number.)
- (c)  $((3.24*6.791)-14.7)/(4.5+82/3)$

Mathematica will not accept brackets or braces as computational grouping symbols. Try  $[(3.24 * 6.791) - 14.7]/(4.5 + 82/3)$ . Do you get the same answer?)

- (d)  $2.6^2.99$  (The symbol  $\wedge$  is the power sign.)
- (e)  $3 5$  (Don't forget the space between 3 and 5.)

5. All built-in Mathematica commands or words begin with a capital letter and use “brackets” if a grouping symbol is appropriate. Type in and evaluate each of the following.
- (a) `Log[12.7]` This is the natural logarithm  $\ln 12.7$ .
  - (b) `E^2.5` This is  $e^{2.5}$ .
  - (c) `Log[10, 72.8]` This is the common logarithm  $\log 72.8$ . Notice that the first number, if listed, indicates the base.
  - (d) `Sqrt[64.0]` This is  $\sqrt{64}$ .
  - (e) `Abs[-2.5]` This is  $|-2.5|$ .
  - (f) `Sin[Pi/6]` This is  $\sin(\pi/6)$ . Trigonometric functions are defined as a function of radians. If a degree unit is used, it must be indicated as shown next.
  - (g) `Cos[30 Degree]` This is  $\cos(30^\circ)$ .
  - (h) `ArcTan[1]` This is  $\tan^{-1}(1)$ .
  - (i) `I*I`  $I$  is  $i = \sqrt{-1}$ .
6. You can find a numerical approximation using the function `N` and `NumberForm`. Type in and evaluate each of the following.
- (a) `E` (This symbol stands for the exact value of  $e$  and will only be replaced with its numerical value if asked.)
  - (b) `N[E]` (Approximate value of  $e$ .)
  - (c) `Pi` (This symbol stands for the exact value of  $\pi$  and will only be replaced with its numerical value if asked.)
  - (d) `N[Pi]` (Approximate value of  $\pi$ .)
  - (e) `Pi//N` (This is another way of asking for the numerical value of the  $\pi$ .)

The function `N` calculates the answer to 16 places but *Mathematica* only displays the default number of places. For viewing the answer to more than the default number of display places but **no more than 16** places you must use the `NumberForm` command along with `N`. Try the following.

- (f) `NumberForm[N[E], 10]`
- (g) `NumberForm[N[Pi], 16]`

You can use the function `N` by itself for calculating and displaying the answer for **more than 17** places. Try the following.

- (h) `N[Pi, 17]` (Did you get one more digit than the last part?)
- (i) `N[Sqrt[2], 50]` (This gives the value of the irrational number  $\sqrt{2}$  to 50 places.)

7. You can also input mathematics in a more standard form using palettes. A basic palette is usually present on the top right hand side of the screen. (If it is not, you can get it by clicking consecutively on the buttons Files, Palettes, and BasicInput. In some types of computers, you may need to click first on Notebook, then Palettes, and finally BasicInput.) You can get a form by clicking on it in the palette. Then you can fill it in by clicking on each location and typing. Use the palette to type in and evaluate each of the following.

(a)  $3^{5.2}$

(b)  $\sqrt{37.4}$

(c)  $\sqrt{3.1^{2.7}}$

(d)  $\frac{125.3}{72}$

(e) The numerical value of  $\sqrt{\frac{5}{9}}$

8. The % key allows you to call back a previous output line. To call back the output from the previous line enter %, two lines up %% , 3 lines up %%%, etc., or better yet, identify the number of the line (say line 6) and enter %6. Evaluate the following.

(a) `N[E^2]`

(b) `ln(e^2)` (You can simply input `Log[%]`). Is the answer correct? Try it again by inputting `Log [%*]`, where \* is replaced by the number of the input line of `N[E^2]`. (Do not put any space between % and the input line number \*)

9. Mathematica can work symbolically to perform algebraic operations. Try these.

(a) `Expand[(a+1)^5]`

(b) `Simplify[3x^2-x-9+x^2+7x+5]`

(c) `Factor[x^4 - 1]`

(d) `Together[2/(x-3)-1/(x+2)]`

(e) `Apart[5/((x-3)(x+2))]`

(f) `Cancel[(x^2-2x-3)/(x^2-9)]`

10. Replacement. A single equal sign indicates replacements. Type in the following and then evaluate them by pushing the **Enter** key.

$$x = 5; y = 12; z = a + b;$$

Enter all of them in just one cell; you may enter each on a separate line or all on one line. If you type in more than one on the same line, be sure to separate them by semicolons. The final semicolon in a line suppresses the final output. If you have used all the semicolons, you should not get any output! Now evaluate the following.

(a) `x` (This should give the assigned value of  $x$  which is 5.)

10. Continued.

(b) `z` (This should give the expression that  $z$  stands for:  $a + b$ .)

(c) `x y^2` (Remember to insert the space between  $x$  and  $y$  to indicate a product.)

(d) `Expand[z^x]` (This is  $(a + b)^5$ .)

11. The assignments for  $x$ ,  $y$  and  $z$  are permanent; this means that to reuse these variables for something else requires that they first be “cleared”. Type in the following and evaluate it.

(a) `Clear[x,y,z]`

(b) `x` (You should get  $x$  and not its previous value of 5.)

It is a good idea to clear each variable after its use, otherwise its reuse will give unintended results.

12. The `Solve` function can be used to solve systems of equations. Input each part in one cell and evaluate it.

(a) `Solve[{x-2y==4,x-1==5y}, {x,y}]`

This is the  $2 \times 2$  linear system of equations:  $x - 2y = 4$  and  $x - 1 = 5y$ . To solve two or more equations simultaneously, enter the equations and the variables you want to solve for as lists. In Mathematica’s syntax a list is any collection of terms separated by commas and grouped by braces.

(b) `Solve[{3x-4y==4,x+2y==7}, {x,y}]`

(c) `f = 2x+y; g = 5x+2y;`

(d) `Solve[{f==2, g==3},{x,y}]` (This solves the system of equations  $2x + y = 2$  and  $5x + 2y = 3$ .)

(e) `Clear[f,g]`

13. It might not be possible or even desirable to find exact solutions to certain equations. You can find numerical approximation of solutions of most equations using the `NSolve` function. Type in and evaluate each of the following.

(a) `NSolve[2x-3==x^2-3x-4]`

(b) `NSolve[2x-3==x^2-3x-4, x]` (Did you get the same answer as in part a?)

(c) `NSolve[{x^2+y^2==16, y==x^2-2x+2}, {x,y}]`

14. The equal sign for entering functions is `:=` (not just `=`). Its independent variable is indicated by an underline (`shift` `⏎`) after it. Both are placed in one bracket pair. Do not use capital letters in naming functions since it might get mixed up with Mathematica’s reserved functions. Enter the following functions and then perform the indicated steps.

`f[x_]:=x^2 - 5; g[x_] :=Log[x]/x` (The functions defined are  $f(x) = x^2 - 5$  and  $g(x) = \frac{\ln x}{x}$  and the independent variable for both is  $x$ . Here, a semicolon at the end of the line has no effect!)

14. Continued.

- (a) `f[3]`
- (b) `g[f[x]]`
- (c) `g[f[3]]`
- (d) Obtain a numerical approximation for part c. (Type in `N[g[f[3]]]` or `N[%]` line # of part (c).)
- (e) `Solve[f[x]==4]` This solves the equation  $x^2 - 5 = 4$
- (f) `Clear[f,g]`

15. You can draw a function using the `Plot`, `ParametricPlot`, `Plot3D` and `ParametricPlot3D` commands. The Mathematica syntax requires stating the independent variable(s) and the domain (minimum and maximum values of each independent variable). Enter the following.

- (a) `Plot[x^2-3x+4,{x,-10,10}]` (This gives the graph of  $y = x^2 - 3x + 4$  from  $x = -10$  to  $x = 10$ .)  
Redraw using the domain from  $-12$  to  $8$  and then again from  $-0.1$  to  $1.2$ .
- (b) `Plot[Log[Abs[x]],{x,-6,6}]` (This gives the graph of  $y = \ln|x|$  from  $x = -6$  to  $x = 6$ . You might see an error message since for  $x = 0$ ,  $\ln|x|$  is undefined.)
- (c) Mathematica can plot parametric equations, in two or three dimensions. Try `ParametricPlot3D[{Cos[t],Sin[t],t},{t,0,10}]` This is the graph of an ascending helix.
- (d) Mathematica can plot 3-dimensional graphs of the form  $z = f(x,y)$ .  
`Plot3D[Sin[x^2 y],{x,-Pi,Pi},{y,-Pi,Pi}]` This is the graph of  $z = \sin(x^2y)$ .

16. You can read the coordinates of the point on a graph by first clicking on the graph so a rectangular box appears around it, and next holding down the `Ctrl` key (the `Alt` key or equivalent on some machines) while moving the cursor over it. The coordinates will appear in the lower bar of your notebook. Try the following.

- (a) `f[x_]:=-x^2+4x+5; Plot[f[x],{x,-2,4}]`
- (b) Click on the graph of part a. Hold down the `Ctrl` key (the `Alt` key or equivalent on some machines) and move the cursor over to a point on the graph with  $x$ -coordinate equal to 1. What is the  $y$ -coordinate of this point? (1, ?)

17. You can plot two or more functions on the same coordinate system. Try the following.

- (a) `g[x_]:=E^(x); Plot[{f[x],g[x]},{x,-2,4}]`
- (b) Read off the coordinates of their two points of intersections. (Click on the graph, hold down the `Ctrl` key (the `Alt` key or equivalent on some machines) and move the cursor over to points of intersection)
- (c) `Clear[f,g]`

18. We can use Mathematica to evaluate one dimensional limits. Enter the following `Limit[Sin[x]/x,x->0]` This evaluates  $\lim_{x \rightarrow 0} (\sin x/x)$ .

Mathematica can also evaluate one dimensional left and right limits. But it can not evaluate two or higher dimensional limits. Also, Mathematica tends to find the right limit only and not check whether the left limit is the same value as the right limit. Therefore, if the limit does not exist, Mathematica will probably give the wrong answer.

19. Mathematica can do symbolic differentiation. Input the following.

(a) `Clear[f,x,y,z]`

This erases the old  $f$  function, if it exists, and the old values of  $x$ ,  $y$ , and  $z$ , if any.

(b) `f[x_]:=x^3-2x^2+5;f'[x]` Is  $f'(x) = -4x + 3x^2$ ?

(c) `Clear[f];f[x_,y_,z_]:=x^4 z^3 y +x Sin[z+y];  
D[f[x,y,z],x]`

This is the first partial derivative of  $f$  with respect to  $x$ , this can also be done by `D[f[x,y,z],{x,1}]`.

(d) `D[f[x,y,z],{x,2},{y,1},{z,1}]` This is  $\frac{\partial^3 f}{\partial z \partial y \partial^2 x}$ .

20. Mathematica does symbolic integration. Mathematica evaluates definite integrals both numerically, and, also, by doing symbolic integration first. Input the following.

(a) `Integrate[x^2+1,x]` This evaluates  $\int (x^2 + 1) dx$ .

(b) `Integrate[x^2+1,{x,-1,2}]` This evaluates  $\int_{-1}^2 (x^2 + 1) dx$ .

(c) `Clear[f];f[x_,y_]:=1-(x^2/4)-(y^2/9);  
Integrate[f[x,y],{x,-2,2},{y,-3,3}]`

This evaluates the iterated integral  $\int_{-2}^2 \int_{-3}^3 f dx dy$  which gives the volume under the surface  $z = 1 - \frac{x^2}{4} - \frac{y^2}{9}$  and above the  $xy$ -plane.

(d) Numerical Integration. Often we do not know the antiderivative of a function but `NIntegrate[Exp[-x^2],{x,0,Infinity}]` This evaluates  $\int_0^{\infty} e^{-x^2} dx$ .

21. It is essential, both as a courtesy to future users, and to continued problem-free usage, to leave the computers as you find them. When you are done, close the software being used and/or log off properly.