Mathematica Commands For Matrices

1. Braces, { }, signal a "list" to Mathematica. A list can be used to represent a set, an interval, a table of data, the parameters in a command, a matrix, etc. Now try the following.

   (a) `list1={-3, 2/5, 1.3, Pi, 8, 5}`

   (b) `ListPlot[list1]`
   Notice that the values of list1 are used as the y values and each is matched with a x value according to the order it appear in the list starting from $x = 1$.

   (c) `list2={{2,-3}, {-1,2/5}, {0,1.3}, {1,Pi}, {4.5,8}, {3,5}}`

   (d) `ListPlot[list2]` This time each element of the list is an ordered pair and graphed as such.

2. It is possible to combine lists to generates new ones. In particular, one can input the data for each variable separately, and then combine the two. Let's reproduce list2 by combining list1 with another list.

   (a) `list3={2, -1, 0, 1, 4.5, 3}`

   (b) `list4={list3, list1}`
   Notice that list4 includes both list1 and list3 but appropriate values are not paired up.

   (c) `list5=Transpose[list4]`
   The command `Transpose` matches up corresponding elements of list3 and list1, which makeup list4. (Actually, it is a 6-row, 2-column matrix that we interpret as ordered pairs.)

   (d) `list5==list2` A true output confirms equality of these two lists.

   (e) `Clear[list1, list2, list3, list4, list5]`

3. You can input a matrix as a list by inputting each row as a list in the order they appear in the matrix. You can add or subtract matrices using $+$ and $-$ signs. The symbol "." is the multiplication sign for matrices.

   (a) `m1={{1,2}, {3,4}}` This is the matrix $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$.

   (b) `TableForm[m1]` The matrix $m1$ is listed as in a spreadsheet (table form).

   (c) `MatrixForm[m1]` The matrix $m1$ is displayed in its standard matrix form.

   (d) `m2={{5,6}, {7,8}}` This is the matrix $\begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix}$.

   (e) `s=m1+m2` s is the sum of $m1$ and $m2$ which is obtained by adding their corresponding elements.

   (f) `m1-2*m2` This is $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} - 2\begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} - \begin{pmatrix} 10 & 12 \\ 14 & 16 \end{pmatrix} = \begin{pmatrix} -9 & -10 \\ -11 & -12 \end{pmatrix}$.

   (g) `p=m1.m2` This gives the product of matrices $m1$ and $m2$.

   (h) `MatrixForm[p]`

3. Continued

   (i) `m1cubed=MatrixPower[m1,3]` The command `MatrixPower` gives the indicated power of a matrix. In this case $m1^3$.

   (j) `m1inv=Inverse[m1]` The command `Inverse` gives the inverse of the matrix.

   (k) `m1inv.m1` Remember that the product of a matrix and its inverse is the identity matrix.

   (l) `tm2=Transpose[m2]` The `Transpose` command interchanges rows and columns of the matrix.

   (m) `MatrixForm[tm2]` Compare this matrix with the $m2$ matrix.

   (n) `Clear[m1, m2, s, p, m1inv, tm2]`

4. A linear system of equations $AX = B$can be solved using inverse of the coefficient matrix or using `Solve` or `NSolve` commands. Let's try these two methods for the example

$$x + 2y - 2z = -1$$
$$x + 3y - z = 1$$
$$x + 2y - z = 3$$

   (a) `mA={{1,2,-2}, {1,3,-1}, {1,2,-1}}; mX={x,y,z}; mB={-1,1,3};`

   (b) Input each `MatrixForm` in a different cell and inspect them to verify correctness of data entry.
   `MatrixForm[mA]`
   `MatrixForm[mX]`
   `MatrixForm[mB]`

   (c) `mInvA=Inverse[mA]`

   (d) `mInvA.mB` The x is the first value, y the second and z the last.

   (e) `NSolve[mA.mX==mB, mX]` Did you get the same answer?

   (f) `Clear[mA, mX, mB, mInvA]`

5. We can find the determinant, eigenvalues and eigenvectors of a square matrix as follows.

   (a) `mA={{3,-1,0,0}, {1,1,0,0}, {0,0,-2,6},{0,0,-3,4}};`

   (b) `MatrixForm[mA]`

   (c) `Det[mA]` This gives the determinant of mA.

   (d) `Eigenvalues[mA]` This gives the eigenvalues of mA. Notice that this matrix has a pair of complex conjugate eigenvalues and a repeated real eigenvalue.

   (e) `Eigenvectors[mA]` This gives the eigenvectors of mA. Notice that this matrix has a pair of complex conjugate eigenvectors and only one eigenvector corresponding to the real eigenvalue.

   (f) `Eigensystem[mA]` This gives both eigenvalues and eigenvectors of mA. The first list is the list of eigenvectors and the second list is the list corresponding eigenvectors.

   (g) `Clear[mA]`