Math 3280 HOMEWORK #4 Name:

Due 2/7/2025, 11:30 a.m.

Solve the following problems and staple your solutions to this cover sheet. (Computer outputs must be put in the appropriate place in the solution, not attached as an appendix. You may physically cut and paste the output in the problem or allow appropriate space in the printout to add your hand written work.)

1. Consider vectors $\vec{x_1}(t) = \begin{bmatrix} e^{2t} \\ 0 \end{bmatrix}$ and $\vec{x_2}(t) = \begin{bmatrix} t\,e^{2t} \\ e^{2t} \end{bmatrix}$. Show $\vec{x_1}(t)$ and $\vec{x_2}(t)$ are solutions of the system $\vec{x'}(t) = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix} \vec{x}(t)$. Find vectors $\vec{x_1}(0)$ and $\vec{x_2}(0)$ and show they are linearly indpendent.

2. Consider vectors $\vec{x_1}(t) = \begin{bmatrix} e^{3t} \\ e^{3t} \end{bmatrix}$ and $\vec{x_2}(t) = \begin{bmatrix} (t+0.5)\,e^{3t} \\ t\,e^{3t} \end{bmatrix}$. Show $\vec{x_1}(t)$ and $\vec{x_2}(t)$ are solutions of the system $\vec{x'}(t) = \begin{bmatrix} 5 & -2 \\ 2 & 1 \end{bmatrix} \vec{x}(t)$. Find vectors $\vec{x_1}(0)$ and $\vec{x_2}(0)$ and show they are linearly indpendent.

3. Find the general solution of $\vec{x'}(t) = \begin{bmatrix} 1 & 3 \\ 1 & -1 \end{bmatrix} \vec{x}(t)$. Note: You may use the results of the last HW.

4. Find the general solution of $\vec{x'}(t) = \begin{bmatrix} 2 & 1 \\ -1 & 0 \end{bmatrix} \vec{x}(t)$. Note: You may use the results of the last HW.

5. Find the general solution of $\vec{x'}(t) = \begin{bmatrix} 2 & 2 \\ -1 & 0 \end{bmatrix} \vec{x}(t)$. Note: You may use the results of the last HW.

6. Find the general solution of $\vec{x'}(t) = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \vec{x}(t)$. Note: You may use the results of the last HW.

7. Write Chap 2, Exer 2(c) in the form $\vec{x'} = A\vec{x}$ and find its general solution. Note: You may use Mathematica to find eigenvalues and eigenvectors. Don't draw its phase portrait.

8. Write Chap 2, Exer 2(d) in the form $\vec{x'} = A\vec{x}$ and find its general solution. Note: You may use Mathematica to find eigenvalues and eigenvectors. Don't draw its phase portrait.

9. Find a general solution $x = x(t)$ of the 2nd order linear constant coefficient ODE $x'' + 5x' - 6x = 0$. Note: Consider its characteristic equation. Do not convert it to a planar system.

10. Convert the 2nd order linear constant coefficient ODE $x'' + 5x' - 6x = 0$ to a planar system. Solve the planar system. Compare your solution $x(t)$ with your answer in the above problem.