Math 3280 HOMEWORK # 11 Name:

Due 4/18/2025, 11:30 a.m., before start of the class

Solve the following problems and staple your solutions to this cover sheet. (Computer outputs must be put in the appropriate place in the solution, not attached as an appendix. You may physically cut and paste the output in the problem or allow appropriate space in the printout to add your hand written work.)

For problems 1 & 2, draw the phase line portraits of each first order ODE and determine if the given pair are qualitatively equivalent or not.

1. $\frac{dx}{dt} = -1 + 5x - x^2$, $\frac{dy}{dt} = y(1 - y)$
2. $\frac{dx}{dt} = x(1 + x^2)$, $\frac{dy}{dt} = y(1 - y^2)$

For problems 3-5, (a) Discuss the phase line portrait structure of each 1st order ODE, (b) Determine all bifurcation points, if any, and draw the bifurcation diagram, and (c) Classify the type of bifurcation, if possible.

3. $\frac{dx}{dt} = x^2 - \mu^2$
4. $\frac{dx}{dt} = x(x^2 - 1 - \mu)$
5. $\frac{dx}{dt} = \mu x + x^3 - x^5$

For $\frac{dx}{dt} = f(x, \mu)$ we can use graphs of $y = f(x, \mu)$ as μ varies to classify the critical points and draw the phase line portraits. You can more easily do this using animation by applying Manipulate (or Animate) command of Mathematica. See below. The Manipulate command gives you easier manual control! The books asks you to use animation in the following two problems.

6. Chap 6, Exer 1(a). Note: Show the animation command (Manipulate or Animate) along with one (or more) graphs.
7. Chap 6, Exer 1(b). Note: Show the animation command (Manipulate or Animate) along with one (or more) graphs.
8. Chap 6, Exer 4(a).
9. Chap 6, Exer 4(b).
10. Free Points.

Mathematica Command

To animate graphs of $y = f(x, \mu)$, displayed for $a \le x \le b$ and $c \le y \le d$, for μ values between s and t, try

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Manipulate[Plot[f(x, μ), {x, a, b}, PlotRange->{{a,b},{c,d}}], {μ, s, t}]

Animate[Plot[f(x, μ), {x, a, b}, PlotRange->{{a,b},{c,d}}], {μ, s, t}]
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