Math 3280 HOMEWORK #9 Name:

Due 3/22/2024, 11:30 a.m., before start of the class

Solve the following problems and staple your solutions to this cover sheet. (Computer outputs must be put in the appropriate place in the solution, not attached as an appendix. You may physically cut and paste the output in the problem or allow appropriate space in the printout to add your hand written work.)

1. Chap 3, Exer 3.

In the next three problems you are asked to consider the different cases for Exercise 4 of Chapter 3 for $\mu > 0$. Note: We have investigated the long-term behavior of competition models extensively in class. Find and classify the critical points and consider the phase portrait. You may also consider the graphs of the system $2 - x - y = 0$, $\mu - y - \mu^2 x = 0$.

2. Chap 3, Exer 4, for $0 < \mu < \frac{1}{2}$.
3. Chap 3, Exer 4, for $\frac{1}{2} \leq \mu < 1$ and $1 \leq \mu < 2$.
4. Chap 3, Exer 4, for $\mu > 2$.
5. Chap 3, Exer 5(a). Note: Also answer the questions after part b for this part.
6. Chap 3, Exer 5(b). Note: Change the second equation to $y' = y(3 - x - 2y)$ to make this exercise different than exercise 2 in chapter 3. Equilibrium points are O = (0, 0), C = (0, 1.5), D = (2, 0), and E = (1, 1). Also answer the questions after part b for this part.
7. Chap 3, Exer 7.
8. It can be shown that the average values of the prey and predator population sizes for models of the form $x' = x(a - by)$ and $y' = y(-c + dx)$ are the corresponding values of the positive critical point. Reconsider exercise 2 in chapter 3 with an added harvesting represented by h is the system below.

$$\begin{aligned} \frac{dx}{dt} &= x(2-y) - hx \\ \frac{dy}{dt} &= y(-3+x) - hy \end{aligned}$$

   Draw the phase portrait of this system for $h = 0.5$. State the average values of the prey and predator population sizes for the exercise 2 in chapter 3 and this problem. What effect does a moderate amount of harvesting have on the populations?

9. Draw the phase portrait of the following predator-prey system in which the prey is not the only food source of the predator.

$$\begin{aligned} \frac{dx}{dt} &= x(1-0.5y) \\ \frac{dy}{dt} &= y(0.5 - y + 0.25x) \end{aligned}$$

   What happens to prey and predator populations in the long term?

10. Free Points!

Mathematica Commands

Consider the autonomous planar system $x'(t) = ax + by$ and $y'(t) = cx + dy$.

Graph the direction field as follows.

```
VectorPlot[{a x + b y, c x + d y}, {x, xmin, xmax}, {y, ymin, ymax},
                                                   VectorScaling->Automatic]
```

(`xmin, xmax, ymin,` and `ymax` are the lower and uper range of values of x and y, respectively.)

You can make all vectors the same length, if desired, by dropping the command `VectorScaling->Automatic`.

```
graph1=VectorPlot[{a x + b y, c x + d y}, {x, xmin, xmax},{y, ymin, ymax}]
```

To find a solution as a function of the initial value $x(0) = x_0$ and $y(0) = y_0$ do the following.

```
gensol[x0_, y0_]:=NDSolve[{x'[t]==a x[t]+b y[t], y'[t]==c x[t]+d y[t],
                          x[0]==x0, y[0]==y0}, {x[t], y[t]}, {t, tmin, tmax}]
```

(`tmin` and `tmax` are the lower and uper range of values of t. Often, it is helpful to use a negative value for `tmin`.)

Then you can find several solutions for the different initial values (x_0, y_0): `(x1, y1); (x2, y2);` ...; `(xn, yn)`.

```
sol[1]=gensol[x1, y1]; sol[2]=gensol[x2, y2]; ··· ; sol[n]=gensol[xn, yn];
```

These solutions can be graphed on the same coordinate system using `ParametricPlot` and `Table` commands.

```
graph2=ParametricPlot[Evaluate[Table[{x[t],y[t]}/.sol[i], {i,n}]],
                     {t, tmin, tmax},PlotRange->{{xmin, xmax}, {ymin, ymax}}]
```

(n is the number of solutions and `{i, n}` steps `i` from 1 to `n` in steps of one. `{x[t], y[t]}/.sol[i]` replaces $(x(t), y(t))$ with the `i`th solution, `sol[i]`.

Now, to display the vector field and the trajectories on the same coordinate system do the following.

```
Show[{graph1, graph2}] .
```