Math 3280 HOMEWORK #8 Name:

Due 3/15/2024, 11:30 a.m., before start of the class

Solve the following problems and staple your solutions to this cover sheet. (Computer outputs must be put in the appropriate place in the solution, not attached as an appendix. You may physically cut and paste the output in the problem or allow appropriate space in the printout to add your hand written work.)

1. Chap 2, Exer 4(e). Determine if critical points are hyperbolic. For non-hyperbolic critical points, Hartman's Linearization Theorem can not be applied. Find the general solution and graph several trajectories on the same coordinate system, using the ContourPlot command of Mathematica.

2. Chap 2, Exer 4(g). Determine if critical points are hyperbolic. For non-hyperbolic critical points, Hartman's Linearization Theorem can not be applied. Find the general solution and graph several trajectories on the same coordinate system, using the ContourPlot command of Mathematica.

Apply the following combined solution technique in problems 3 & 4.

Combined Solution: Analyze the system by combining analytical and numerical techniques, as follows. Find all classify all critical points, if possible. Consider invariant lines or curves, stable and unstable manifolds, and local behavior of solutions near hyperbolic equilibrium points. If a critical point is not hyperbolic, consider analyzing it other ways, for example, by solving the system. Use Mathematica to graph the phase portrait (vector field and several representative trajectories on the same coordinate system). Use appropriate initial points for best representation of the trajectories.

3. Draw the phase portrait of

$$\begin{aligned}\frac{dx}{dt} &= -2y \\ \frac{dy}{dt} &= x(x^2-1)+2y\end{aligned}.$$

4. Draw the phase portrait of

$$\begin{aligned}\frac{dx}{dt} &= x - xy \\ \frac{dy}{dt} &= 2y - 2xy\end{aligned}.$$

5. Chap 2, Exer 7.

6. Chap 2, Exer 8.

7. Chap 3, Exer 1.

8. Chap 3, Exer 2.

9. Free Points!

10. Free Points!

Mathematica Commands

Consider the autonomous planar system $x'(t) = ax + by$ and $y'(t) = cx + dy$.

Graph the direction field as follows.

```
VectorPlot[{a x + b y, c x + d y}, {x, xmin, xmax}, {y, ymin, ymax},
                                                  VectorScaling->Automatic]
```

(`xmin, xmax, ymin,` and `ymax` are the lower and uper range of values of x and y, respectively.)

You can make all vectors the same length, if desired, by dropping the command `VectorScaling->Automatic`.

```
graph1=VectorPlot[{a x + b y, c x + d y}, {x, xmin, xmax},{y, ymin, ymax}]
```

To find a solution as a function of the initial value $x(0) = x_0$ and $y(0) = y_0$ do the following.

```
gensol[x0_, y0_]:=NDSolve[{x'[t]==a x[t]+b y[t], y'[t]==c x[t]+d y[t],
                          x[0]==x0, y[0]==y0}, {x[t], y[t]}, {t, tmin, tmax}]
```

(`tmin` and `tmax` are the lower and uper range of values of t. Often, it is helpful to use a negative value for `tmin`.)

Then you can find several solutions for the different initial values (x_0, y_0): `(x1, y1); (x2, y2); ...; (xn, yn)`.

```
sol[1]=gensol[x1, y1]; sol[2]=gensol[x2, y2]; ··· ; sol[n]=gensol[xn, yn];
```

These solutions can be graphed on the same coordinate system using `ParametricPlot` and `Table` commands.

```
graph2=ParametricPlot[Evaluate[Table[{x[t],y[t]}/.sol[i], {i,n}]],
                      {t, tmin, tmax},PlotRange->{{xmin, xmax}, {ymin, ymax}}]
```

(n is the number of solutions and `{i, n}` steps `i` from 1 to `n` in steps of one. `{x[t], y[t]}/.sol[i]` replaces $(x(t), y(t))$ with the `i`th solution, `sol[i]`.

Now, to display the vector field and the trajectories on the same coordinate system do the following.

```
Show[{graph1, graph2}] .
```