Math 3280 HOMEWORK #7 Name:

Due 2/23/2024, 11:30 a.m., before start of the class

Solve the following problems and staple your solutions to this cover sheet. (Computer outputs must be put in the appropriate place in the solution, not attached as an appendix. You may physically cut and paste the output in the problem or allow appropriate space in the printout to add your hand written work.)

The following are instructions for analytical and numerical solutions.

Analytical Solution: Find all critical points. Find hyperbolic critical points and classify them. Draw the phase portrait by hand by going the following. Draw curves on which $x'(t) = 0$ or $y'(t) = 0$. These curves divide the phase plane into several regions. In each region draw one or more representative direction vector by considering the signs of x' and y'. Using the direction vectors and local behavior of solutions near a hyperbolic critical point draw some trajectories. It might be useful to also consider isoclines, invariant lines or curves (for example, solutions of the form $(x, y) = (0, y)$), if any, and stable and unstable manifolds, if possible. If having difficulty, consider the numerical solution.

Numerical Solution: Use Mathematica to graph the phase portrait (vector field and several representative trajectories on the same coordinate system). Use appropriate initial points for best representation of the trajectories. The information gained from the analytical solution can be helpful for this. See below, first homework, or your book for Mathematica commands.

1. Chap 2, Exer 4(a). Draw the phase portrait analytically.
2. Chap 2, Exer 4(a). Draw the phase portrait numerically. Do your analytical and numerical work agree?
3. Chap 2, Exer 4(b). Draw the phase portrait analytically.
4. Chap 2, Exer 4(b). Draw the phase portrait numerically. Do your analytical and numerical work agree?
5. Chap 2, Exer 4(c). Draw the phase portrait analytically.
6. Chap 2, Exer 4(c). Draw the phase portrait numerically. Do your analytical and numerical work agree?
7. Chap 2, Exer 4(d). Draw the phase portrait analytically.
8. Chap 2, Exer 4(d). Draw the phase portrait numerically. Do your analytical and numerical work agree?
9. Free Points!
10. Free Points!

Mathematica Commands

Consider the autonomous planar system $x'(t) = ax + by$ and $y'(t) = cx + dy$.

Graph the direction field as follows.

```
VectorPlot[{a x + b y, c x + d y}, {x, xmin, xmax}, {y, ymin, ymax},
                                                    VectorScaling->Automatic]
```

(`xmin, xmax, ymin,` and `ymax` are the lower and uper range of values of x and y, respectively.)

You can make all vectors the same length, if desired, by dropping the command `VectorScaling->Automatic`.

```
graph1=VectorPlot[{a x + b y, c x + d y}, {x, xmin, xmax},{y, ymin, ymax}]
```

To find a solution as a function of the initial value $x(0) = x_0$ and $y(0) = y_0$ do the following.

```
gensol[x0_, y0_]:=NDSolve[{x'[t]==a x[t]+b y[t], y'[t]==c x[t]+d y[t],
                           x[0]==x0, y[0]==y0}, {x[t], y[t]}, {t, tmin, tmax}]
```

(`tmin` and `tmax` are the lower and uper range of values of t. Often, it is helpful to use a negative value for `tmin`.)

Then you can find several solutions for the different initial values (x_0, y_0): `(x1, y1); (x2, y2);` $\ldots$`; (xn, yn)`.

```
sol[1]=gensol[x1, y1]; sol[2]=gensol[x2, y2]; ··· ; sol[n]=gensol[xn, yn];
```

These solutions can be graphed on the same coordinate system using `ParametricPlot` and `Table` commands.

```
graph2=ParametricPlot[Evaluate[Table[{x[t],y[t]}/.sol[i], {i,n}]],
                      {t, tmin, tmax},PlotRange->{{xmin, xmax}, {ymin, ymax}}]
```

(n is the number of solutions and `{i, n}` steps `i` from 1 to `n` in steps of one. `{x[t], y[t]}/.sol[i]` replaces $(x(t), y(t))$ with the `i`th solution, `sol[i]`.

Now, to display the vector field and the trajectories on the same coordinate system do the following.

```
Show[{graph1, graph2}] .
```