Math 3280 HOMEWORK #6 Name:

Due 2/16/2024, 11:30 a.m., before start of the class

Solve the following problems and staple your solutions to this cover sheet. (Computer outputs must be put in the appropriate place in the solution, not attached as an appendix. You may physically cut and paste the output in the problem or allow appropriate space in the printout to add your hand written work.)

1. Chap 2, Exer 2(d). Write in the form $X' = AX$. Find the general solution. Discuss solutions of this system and draw several representative trajectories in the phase plane by hand. Classify the critical point. Note: You may use Mathematica to find eigenvalues and eigenvectors.

2. Chap 2, Exer 2(d). Use Mathematica to graph its phase portrait (vector field and several representative trajectories on the same coordinate system). See below, the first homework, or your book for Mathematica commands.

3. Chap 2, Exer 2(f). Write in the form $X' = AX$. Classify the critical point. Find the general solution. Draw several representative trajectories in the phase plane by hand. Note: You may use Mathematica to find eigenvalues and eigenvectors. Then use Mathematica to graph its phase portrait (vector field and several representative trajectories on the same coordinate system).

4. Chap 2, Exer 2(e). Write in the form $X' = AX$. Classify the critical point. Find the general solution. Draw several representative trajectories in the phase plane by hand. Note: You may use Mathematica to find eigenvalues and eigenvectors. Then use Mathematica to graph its phase portrait (vector field and several representative trajectories on the same coordinate system).

5. Consider $X'(t) = AX$ with $A = \begin{bmatrix} -3 & -1 \\ 2 & 0 \end{bmatrix}$. Classify the critical point. Find its general solution. Use Mathematica to graph its phase portrait (vector field and several representative trajectories on the same coordinate system). You may use Mathematica to find eigenvalues and eigenvectors.

6. Chap 2, Exer 3 (a, b(i)). Classify the critical point. You may use Mathematica.

7. Chap 2, Exer 3 (b(ii)). Classify the critical point. You may use Mathematica.

8. Chap 2, Exer 3 (b(iii)). Classify the critical point. You may use Mathematica.

9. Chap 2, Exer 3 (b(iv)). Classify the critical point. You may use Mathematica.

10. Free points!

Mathematica Commands

Consider the autonomous planar system $x'(t) = ax + by$ and $y'(t) = cx + dy$.

Graph the direction field as follows.

```
VectorPlot[{a x + b y, c x + d y}, {x, xmin, xmax}, {y, ymin, ymax},
                                                  VectorScaling->Automatic]
```

(`xmin, xmax, ymin,` and `ymax` are the lower and uper range of values of x and y, respectively.)

You can make all vectors the same length, if desired, by dropping the command `VectorScaling->Automatic`.

```
graph1=VectorPlot[{a x + b y, c x + d y}, {x, xmin, xmax},{y, ymin, ymax}]
```

To find a solution as a function of the initial value $x(0) = x_0$ and $y(0) = y_0$ do the following.

```
gensol[x0_, y0_]:=NDSolve[{x'[t]==a x[t]+b y[t], y'[t]==c x[t]+d y[t],
                          x[0]==x0, y[0]==y0}, {x[t], y[t]}, {t, tmin, tmax}]
```

(`tmin` and `tmax` are the lower and uper range of values of t. Often, it is helpful to use a negative value for `tmin`.)

Then you can find several solutions for the different initial values (x_0, y_0): `(x1, y1); (x2, y2); ...; (xn, yn)`.

```
sol[1]=gensol[x1, y1]; sol[2]=gensol[x2, y2]; ··· ; sol[n]=gensol[xn, yn];
```

These solutions can be graphed on the same coordinate system using `ParametricPlot` and `Table` commands.

```
graph2=ParametricPlot[Evaluate[Table[{x[t],y[t]}/.sol[i], {i,n}]],
                      {t, tmin, tmax},PlotRange->{{xmin, xmax}, {ymin, ymax}}]
```

(n is the number of solutions and `{i, n}` steps `i` from 1 to `n` in steps of one. `{x[t], y[t]}/.sol[i]` replaces $(x(t), y(t))$ with the `i`th solution, `sol[i]`.

Now, to display the vector field and the trajectories on the same coordinate system do the following.

```
Show[{graph1, graph2}] .
```