Math 3280 HOMEWORK #3 Name:

Due 1/26/2024, 11:30 a.m.

Solve the following problems and staple your solutions to this cover sheet. (Computer outputs must be put in the appropriate place in the solution, not attached as an appendix. You may physically cut and paste the output in the problem or allow appropriate space in the printout to add your hand written work.)

1. Chap 1, Exer 8. Hints: The initial conditions are $\sigma_A(0) = \sigma_B(0) = 0$. Solve for σ_A and use it to solve for σ_B. The mass is $\sigma_B V$.

2. Consider the ODE $x'(t) = x(x-3)(x+1)^2$. Find its equilibrium solutions. Classify the equilibrium solutions two ways: a. Finding the sign of $x'(t)$ and drawing the phase line, and b. Applying the Linearization Theorem.

3. Consider $\frac{dx}{dt} = x(x-1)$. Find its equilibrium points and classify them. Determine the intervals, (initial) x-values, on which the solution $x(t)$ would be increasing or decreasing. Determine the intervals, x-values, on which the solution $x(t)$ would be concave up or down. Use these information to graph several representative flows (solutions) in the tx-plane, by hand. Hints: For $\frac{dx}{dt} = f(x)$, monotonicity of $x(t)$ is determined by the sign of $f(x)$ and its concavity is determined by the sign of $\frac{d^2x}{dt^2} = \frac{d}{dt}(\frac{dx}{dt}) = \frac{d}{dx}(\frac{dx}{dt})\frac{dx}{dt} = f'(x)\,f(x)$.

4. Consider $x'(t) = 2x^3 + x^2 - 4x - 2$. Find its equilibrium points and classify them. Determine the intervals, (initial) x-values, on which the solution $x(t)$ would be increasing or decreasing. Determine the intervals, x-values, on which the solution $x(t)$ would be concave up or down. Use these information to graph several representative flows (solutions) in the tx-plane, by hand. Hints: For $\frac{dx}{dt} = f(x)$, monotonicity of $x(t)$ is determined by the sign of $f(x)$ and its concavity is determined by the sign of $\frac{d^2x}{dt^2} = \frac{d}{dt}(\frac{dx}{dt}) = \frac{d}{dx}(\frac{dx}{dt})\frac{dx}{dt} = f'(x)\,f(x)$.

5. Find the eigenvalue(s) and corresponding eigenvector(s) of the matrix $A = \begin{bmatrix} 1 & 3 \\ 1 & -1 \end{bmatrix}$ both by hand and using Mathematica. See below, the first HW, or your book for Mathematica commands.

6. Find the eigenvalue(s) and corresponding eigenvector(s) of the matrix $A = \begin{bmatrix} 2 & 1 \\ -1 & 0 \end{bmatrix}$ both by hand and using Mathematica. See below, the first HW, or your book for Mathematica commands.

7. Find the eigenvalue(s) and corresponding eigenvector(s) of the matrix $A = \begin{bmatrix} 2 & 2 \\ -1 & 0 \end{bmatrix}$ both by hand and using Mathematica. See below, the first HW, or your book for Mathematica commands.

8. Find the eigenvalue(s) and corresponding eigenvector(s) of the matrix $A = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$ both by hand and using Mathematica. See below, the first HW, or your book for Mathematica commands.

9. Free points!

10. Free points!

Mathematica Commands

Braces, { }, signal a "list" or a row vector to Mathematica.

`List={-3, 2/5, 1.3, Pi}` This is a list of numbers -3, 2/5, 1.3 and π or the row vector $(-3 \;\; 2.5 \;\; 1.3 \;\; \pi)$.

You can input a matrix as a list by inputting each row as a list in the order they appear in the matrix.

`mA={{1,2}},{3,4}}` This is the matrix $mA = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$.

You can find the determinant, eigenvalues and eigenvectors of a square matrix as follows.

`Det[mA]` This gives the determinant of mA.
`Eigenvalues[mA]` This gives the eigenvalues of mA.
`Eigenvectors[mA]` This gives the eigenvectors of mA.
`Eigensystem[mA]` This gives both eigenvalues and eigenvectors of mA. The first list is the list of eigenvectors and the second list is the list of the corresponding eigenvectors.