Math 3280 HOMEWORK #3 Name:

Due 1/23/2026, 11:30 a.m.

Solve the following problems and staple your solutions to this cover sheet. (Computer outputs must be put in the appropriate place in the solution, not attached as an appendix. You may physically cut and paste the output in the problem or allow appropriate space in the printout to add your hand written work.)

1. Consider the ODE $x'(t) = x(x-3)(x+1)^2$. Find its equilibrium solutions and classify them the following two ways.
   a. Finding the sign of $f(x) = x(x-3)(x+1)^2$ and drawing the phase line. Note: Do **NOT** combine the sign of $f(x)$ and the phase line into one graph!
   b. Applying the Linearization Theorem.

Recall that for the autonomous equations $\frac{dx}{dt} = f(x)$, the monotonicity of solutions are determined by the sign of $f(x)$, while the cancavity of solutions is determined by the sign of $\frac{d^2x}{dt^2} = \frac{d}{dt}(\frac{dx}{dt}) = \frac{d}{dx}(\frac{dx}{dt})\,\frac{dx}{dt} = f'(x)\,f(x)$.

2. Consider $\frac{dx}{dt} = x(x-1)$. Find its critical points and classify them. Determine the intervals, (initial) x-values, on which the solution $x(t)$ would be increasing or decreasing. Determine the intervals, x-values, on which the solution $x(t)$ would be concave up or down. Use these information to graph several representative flows (solutions) in the tx-plane, by hand. Note: Classification method is up to you. Pay attention to the flows that change concavity!

3. Consider $x'(t) = 2x^3 + x^2 - 4x - 2$. Find its equilibrium solutions and classify them. Determine the intervals, (initial) x-values, on which the solution $x(t)$ would be increasing or decreasing. Determine the intervals, x-values, on which the solution $x(t)$ would be concave up or down. Use these information to graph several representative trajectories (solutions) in the tx-plane, by hand. Note: Classification method is up to you. Pay attention to the trajectories that change concavity!

4. Consider the differential equation $\frac{dx}{dt} = x^3 - 3x^2 - 9x + 27$. Find and classify its critical points. Determine the intervals, x values, in which the solution $x(t)$ is increasing or decreasing. Determine the intervals, x values, in which the solution $x(t)$ is concave up or concave down. Draw the solution curves with initial values $x(0) = -4,\ -3,\ -2,\ 1,\ 3,\ 4$ in the tx-plane, for $t \geq 0$. Hint: $x^3 - 3x^2 - 9x + 27 = (x-3)^2(x+3)$. Note: Classification method is up to you. Pay attention to the solutions that change concavity!

5. Find the eigenvalue(s) and corresponding eigenvector(s) of the matrix $A = \begin{bmatrix} 1 & 3 \\ 1 & -1 \end{bmatrix}$ both by hand and using Mathematica. See below, the first HW, or your book for Mathematica commands.

6. Find the eigenvalue(s) and corresponding eigenvector(s) of the matrix $A = \begin{bmatrix} 2 & 1 \\ -1 & 0 \end{bmatrix}$ both by hand and using Mathematica. See below, the first HW, or your book for Mathematica commands.

See back of the page!

7. Find the eigenvalue(s) and corresponding eigenvector(s) of the matrix $A = \begin{bmatrix} 2 & 2 \\ -1 & 0 \end{bmatrix}$ both by hand and using Mathematica. See below, the first HW, or your book for Mathematica commands.

8. Find the eigenvalue(s) and corresponding eigenvector(s) of the matrix $A = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$ both by hand and using Mathematica. See below, the first HW, or your book for Mathematica commands.

9. Free points!

10. Free points!

Mathematica Commands

Braces, { }, signal a "list" or a row vector to Mathematica.

`List={-3, 2/5, 1.3, Pi}` This is a list of numbers -3, 2/5, 1.3 and π or the row vector $(-3 \;\; 2.5 \;\; 1.3 \;\; \pi)$.

You can input a matrix as a list by inputting each row as a list in the order they appear in the matrix.

`mA={{1,2}},{3,4}}` This is the matrix $mA = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$.

You can find the determinant, eigenvalues and eigenvectors of a square matrix as follows.

`Det[mA]` This gives the determinant of mA.
`Eigenvalues[mA]` This gives the eigenvalues of mA.
`Eigenvectors[mA]` This gives the eigenvectors of mA.
`Eigensystem[mA]` This gives both eigenvalues and eigenvectors of mA. The first list is the list of eigenvectors and the second list is the list of the corresponding eigenvectors.