EUCLIDEAN AND NON-EUCLIDEAN GEOMETRY

MATH 3120, CRN 11592, Summer 2007

http://faculty.weber.edu/aghoreishi/Math3120_su07/Math3120_su07.asp/

Prerequisite: Math 1220, and either Math 2120 or consent of instructor.

Textbooks: Required Text: Foundations of Geometry, Gerard A. Venema, Pearson Prentice Hall, ISBN 0131437003.
Optional Texts: Another textbook: Roads to Geometries by Wallace and West, Prentice Hall, ISBN 0130413968.
A Book with Metric Approach: Elementary Geometry from an Advanced Standpoint by Edwin E. Moise, Addison Wesley, ISBN 0201508672.
An Experimentalist Book: Experiencing Geometry On Plane and Sphere by David W. Henderson, Prentice Hall, ISBN 0133737705.
A Fun and Informative Bedtime Reading: Journey into Geometries by Marta Sved, The Mathematical Association of America, ISBN 0883855003.
An Excellent Book on the Art of Mathematical Writing: Mathematical Writing by Knuth, Larrabee and Roberts, The Mathematical Association of America, ISBN 088385063X.

Class Meetings:	Lectures:	MTWR 10:30-11:45, Davis Campus, Room 233.
-	Optional Tutoring:	MTWR 11:45-12:15, Davis Campus, Information Commons,
		2nd floor
	Extra Help:	Tutor: Brack Carmony, Information Commons, Hours to be
	_	announced.

Instructor Information: Dr. Afshin Ghoreishi, http://faculty.weber.edu/aghoreishi/, B4 Rm. 505A, Davis Campus Rm. 308K. I will be available MTWR 9:40-10:15 and 11:45-12:15 in the Information Commons. You can also see me by making an appointment.

Procedures: In this course it is assumed that you are familiar with the Euclidean geometry. We will emphasize axiomatic development of geometry, non-Euclidean geometry, and proof and rigor. Our textbook is very well written and you will be assigned to read parts of the book on your own.

I will try to answer a few questions at the beginning of each class, but this time will be limited. We will have homework and two exams. Do not enter the class late & do not come to class if you have to leave early. Turn off pagers, cell phones and other such disruptive devices.

Homework: A problem list and some additional problems are included. You should solve all of them. However, I will collect your best work as follows:

5 problems from each of the chapters 1-4, 7 and 8, and 10 problems from each of chapters 5 and 6, plus problems CP14.1 and CP14.3.

This is an opportunity for you to show off your work. So, choosing the easiest problems or doing sloppy work will not bode well for you! We can discuss all problems in class, as time permits, and also during optional tutoring.

Do not just turn in any solution in the hopes of getting partial credit. At this stage, I expect you to be able to differentiate between good and bad work. To achieve the best solution, expect to do several rewriting! **Turn in each problem in a separate page.** Write only on one side of each page.

Exams: Exams will not have a time limit and will be administered at the Davis Center Testing Center. Exams can be taken anytime during the time periods listed below. **No** make-up exam will be given.

Exam I	July	5-6
Exam II	Aug	1-2

The Davis Testing Center is scheduled to be open 7:30 am - 6:30 pm M-R, 7:30 am - 4:30 pm F, and 7:30 am - 12:00 pm Sat. You must finish an exam within one hour after the closing time. You must also take along a picture I.D. We will meet for our regular lectures during exam days.

Grading: Exams will be curved as needed, but a minimum standard will be retained regardless of the class performance.

Homework	30 percent
Exam I	30 percent
Exam II	40 percent

Other Important Dates:

Independence Day	July	4
Last day to drop with a grade of W	July	20
Pioneer day	July	24

If you decide to drop this class, please inform me of your decision.

Extra Help: I suggest using the Information Commons as our study place. I will be available MTWR 11:45-12:15 for tutoring. In addition, I have arranged for a tutor. His name is Brack Carmony and he will be also be available in the Information Commons.

Course Coverage and Problem List for Math 3120 This list may be adjusted, as needed.				
Chapter	Sections	Problems		
1	All	1.1-1.3, 1.4 (a, b), 1.6, 1.7, 1.10		
2	All	2.1-2.3, 2.5, 2.6, 2.10, 2.11, 2.12 (a, b)		
3	All	3.1, 3.3, 3.5, 3.12, 3.13, AD3.1-3.3		
4	All	4.3-4.5, 4.11, 4.15		
5	All	5.2, 5.4, 5.7, 5.9, 5.11 (not b), 5.13-5.24, 5.26, 5.29-5.36, AD5.1- 5.4, See Hints		
6	All	6.2-6.4, 6.6-6.18, 6.24-6.29, 6.32, 6.33, 6.35, 6.39-6.41, 6.44-6.47, 6.49		
7	7.1-7.5	7.1-7.13		
8	All	8.1-8.12, 8.14-8.18		
13	Parts of 13.1-13.5, Self Reading	None		
14	14.2, Self Reading	CP14.1, CP14.3, Optional: CP14.4, CP14.5		

Additional Problems/Hints

Chapter 3

Axioms:

Consider the following axiomatic system.

Undefined terms: Scorple, Flug

SF1: If A and B are distinct flugs, then A scorples B or B scorples A (the possibility of both happening is not excluded).

- SF2: No flug scorples itself.
- SF3: If A, B, and C (not necessarily distinct) are flugs shuch that A scorples B and B scorples C, then A scorples C.
 SE4. There are consider from the scorples C.
- SF4: There are exactly four flugs.

Prove the following theorems.

AD3.1 - Theorem 1. If a flug scorples another, it is not scorpled by the other.

By Theorem 1, SF1 can be restated as:

SF1*: If A and B are distinct flugs, then A scorples B or B scorples A, but not both.

AD3.2 - Theorem 2. If A scorples B, and C is distinct from A, then A scorples C or C scorples B (possible both).

AD3.3 - Theorem 3. There is at least one flug that scorples every other flug.

Chapter 5

AD5.1 - Let A and B be two distinct points. Let f be a coordinate function for the line containing A and B with f(A) < f(B) . S $A\overline{B} = \{P \in A\overline{B} | f(A) \le f(P) \le f(B)\}$

AD5.2 - Let A and B be two distinct points. Show that there are points C, D and E such that A*C*B, A*B*D and E*A*B. Hint: Consider an appropriate coordinate function.

AD5.3 - Show that if A*B*C ans B*C*D, then A*B*D and A*C*D. Hint: Consider a coordinate function and use the Betweenness Theorem for Points.

AD5.4 - Show that if A*C*D and A*B*C, then A*B*D.

AD5.5 - Given $A\overline{B}$ and distinct points C and D. Show there is a point E such that C*D*E and AB=DE.

AD5.6 - Let \vec{AB} be a ray and let C be a point on it distinct from A and B. Prove $\vec{AB} = \vec{AC}$

Hint: For any two sets S and T, to prove $\hat{S}=T$, show that $S\subseteq T$ and $T\subseteq S$. Another option is to use problem 5.7.

AD5.7 - Let $\angle ABC$ be an angle. Prove that interior $\angle ABC$ is not an empty set if and only if $\mu(\angle ABC) > 0$.

Hints For Chapter 5 Problems

Problem 5.14: Consider a coordinate function and use the Betweenness Theorem for Points.Problem 5.15: If $A \notin \{D, E, F\}$, then A is between two of the vertices of the $\triangle DEF$. WOLOG, assume

D*A*E. If $B \notin D\overline{E}$, then B is on one of the other sides of $\triangle DEF$. WOLOC $B \notin E\overline{F} \setminus \{E\}$ consider a point P between A and B.

Problem 5.16: For any two sets S and T, to prove S=T, show that $S\subseteq T$ and $T\subseteq S$.

Problem 5.17: Consider an appropriate coordinate function.

a

Chapter 6

AD6.1 - State all statements equivalent to Euclidean Parallel Postulate stated in Chap 6.

AD6.2 - Suppose $\angle BAC$ is an angle and point P is on ray AC and it is not point A. Let point Q be the foot of the perpendicular from P onto line AB.

a. Assume $\angle BAC$ is an acute angle and prove that Q is on $A\overline{B}$ and it is not point A.

b. Assume $\angle BAC$ is an obtuse angle and prove that Q is different from A and on the ray opposite to ray \vec{AB} .

AD6.3 - Suppose $A\overline{B}$ $C\overline{D}$ are two distinct segments that share a common midpoint. Prove that \Box ACBD is a parallelogram.

AD6.4 - Prove that if the diagonals of a convex quadrilateral intersect in a point that is the midpoint of each diagonal, then the quadrilateral is a parallelogram.

Textbook Corrections

All corrections apply to the first printing. Corrections designated by * have already being made in the second printing. You probably have a second printing book.

- *Page 7, line 13. The word "to" is missing. (Should be: "...applied to the triangle....")
- 2. Page 13, Exercise 1.2. Change "Moscow papyrus" to "Rhind papyrus."
- 3. *Page 19, line 31. Strike "s" from "questions". (Should be: "...a basic question regarding....")
- 4. *Page 26, line 20. Add "s" to happen. (Should be: "Since this happens for every....")
- 5. *Page 27, line 5. Add a comma after not. (Should be: "other models that do not, shows that....")
- 6. Page 29, line 2 of Exercise 2.1. Change "is" to "it". (Should be: "... it should be possible")
- 7. *Page 29, line 7 of Exercise 2.1. Delete "the table". (Should be: "...lie on to mean resting on.")
- 8. Page 44, line 13. Delete "the". (Should be: "... say that a set")
- 9. Page 47, lines 12 and 15. Change "Theorem 4.2.3" to "Axiom 4.2.3".
- 10. Page 51, line 3 from the bottom in Exercise 4.15. Deltet "and".
- 11. Page 68, line 3. Change "Theorem 5.7.7" to "Corollary 5.7.8."
- 12. *Page 71, first line of statement of Corollary 5.7.2. Change the second C to B. (Should be: "... three points such that B lies on ray AC.")
- 13. Page 74, line 4. Insert "the" between "in" and "interior".
- 14. Page 74, second line of the proof of Corollary 5.7.2. Change Theorem 5.7.7 to Corollary 5.7.7.
- 15. *Page 78, first line of the statement of Theorem 5.7.16. Add the word "if" after "if and only". (Should be: "... if and only if")
- 16. Page 91, Exercise 5.11(b). The Incidence Axiom should be I-3.
- 17. Page 93, Exercise 5.32. Add "Prove that" at the beginning. (Should be: "Prove that if ...").
- 18. Page 109, line 12. Delete "the." (Should be: "... notice that there is")
- 19. Page 109, line 4 of proof of corollary 6.56. Delete "Exercise 6.6.3" to "Exercise 6.3".
- 20. *Page 114, Figure 6.24. The labels are in the wrong size and wrong font.
- 21.*Page 122, second line of Definition 6.8.6. The symbol " \angle " is missing. (Should be: " $\angle BCA \cong \angle EFD$,")22.*Page 127, line 13. The symbol " Δ " is missing. (Should be: " $\Delta ABC \cong \Delta CDA \cong \Delta CFE$, so")
- 23. Page 127, line 17. Change AEGF to $\Box AEGH$.
- *Page 129, line 7. Delete "of". (Should be "...in naming the two special types....")
- *Page 130, line 21. Add an "s" to quadrilateral. (Should be: "...what useful tools these quadrilaterals are.")
- 26. Page 130, statement of Aristotle's Theorem. Change "If $\angle BAC$ is an acute angle..." to "If A, B, and C are three noncollinear points such that $\angle BAC$ is an acute angle...".
- 27. Page 131, line 2 from the bottom. Change "previous paragraph" to "first paragraph of this proof".
- 28. *Page 133, Exercise 6.35, line 2. Add a comma at the end of the line. (Should be: "...if it is not convex, it is called a dart.")

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29.	*Page 133, Exercise 6.35, line 3. The sentence should end with just one period. (Should be: "if it is not convex
30.	it is called a <i>dart</i> .") *Page 135, line 3. Change "remainder" to "entirety." (Should be: "For the entire chapter")
31.	*Page 154, statement of Ceva's Theorem. Change "are concurrent if and only if" to "are concurrent (or are mutually parallel) if and only if" *Page 159, Exercise 7.32(b). Replace "Use Exercise 7.30(b) to prove that" with "If the three Cevian lines are
 33. 34. 35. 	not mutually parallel, then it may be assumed that" Page 166, second line of the second paragraph of the proof of Theorem 8.2.7. Change "∠ BAD" to "∠ BCD". Page 167, 5 lines up from bottom of page. Change Lemma 6.7.8 to Theorem 6.7.8. *Page 167, third line from bottom of page. The symbol "∠" is missing. (Should be: "∠ CC'B' is the supplement
36. 37.	of ∠ <i>AC'B'</i> ,") Page 168, line 3 from the bottom. The reference should be to Definition 6.4.5 rather than to Theorem 6.4.4. *Page 169, line 5. Delete the period (".") before "(RAA hypothesis)."
38	*Page 174, line 9. There is an extra period after the word "is." (Should be: " that is, choose")
39.	*Page 176, last symbols in statement of Theorem 8.4.9. Both primes (') should be under the arrow.
40. 41.	*Page 177, last symbol in statement of Theorem 8.4.12. The prime (') should be under the arrow. Page 180, line 8 Insert "to" between "parallel" and "the".
42. 43.	*Page 187, line 3. The words "the other" should not be repeated. (Should be: " while the other two sides") *Page 192, Exercise 8.15. Change "exits" to "exists." (Should be " there exists a rhombus") *Page 193, Exercise 8.21, Change "decreasing" to "strictly decreasing." (Should be: If $f: (a, b) \rightarrow (c, d)$ is strictly
44.	decreasing and onto") *Page 197, second line from bottom of page. Add "if". (Should be: " if and only if their interiors are disjoint.")
46. 47.	Page 198, line 6. Insert "nonempty" after "each." (Should be: "for each nonempty polygonal region") *Page 198, line 11. There is an extraneous comma after R ₁ . (Should be: "If R is the union of two
48.	nonoverlapping polygonal regions R_1 and R_2 , then") *Page 198, last line. Delete " E ".
49. 50.	Page 230, line 8. Change "at most" to "less than". Page 246, caption for Figure 10.33. Interchange the words "inside" and "outside".
51.	*Page 248, line 16. Add "s" to perimeter. (Should be: " the perimeters of polygons")
52	*Page 248, line 20. Delete "Abu".
53	*Page 256, second line of first full paragraph. Insert "is." (Should be: " this is a single circle")
54	*Page 258, line 1. Delete extraneous "the" at the beginning of the line.
55.	*Page 259, first line of Exercise 10.9. Add "a". (Should be: "an altitude of a triangle is a line")
56	*Page 260, line 6 of Exercise 10.28. Change "MacLauren" to "Maclaurin."
57	*Page 288, line 6 of Proof of Theorem 12.2.7. Change "if" to "it." (Should be: " it follows that")
58.	Page 291, line 9 Change "refections" to "reflections".
59.	*Page 294, line 14. Change "rotation" to "reflection". (Should be: "The reflection and half-turn are different in that")
60.	*Page 300, 7 lines from bottom of page. Add "to". (Should be: " we move on to the proof")
61.	*Page 301, 6 lines from bottom of page. Add "be". (Should be: " proof can be used")
62.	*Page 303, line 20. Delete "we". (Should be: " it could be written as")
63.	*Page 303, line 27. Delete "s" from "translations". (Should be: " and translation theorems.")
64.	*Page 306, line 2 of footnote. Add period after "assumed".

- 65. Page 308, 4 lines up from bottom of page. Change Δ to \angle . (Should be: $\angle ABC \cong \angle ACB$.)
- 66. Page 312, Figure 12.19. The Q and Q' in left hand diagram should be interchanged.
- 67. Page 318, line 14. The symbol \angle is missing. (Should be: $\angle BP'R' \cong \angle RPP'$.)

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- 68. *Page 318, second line from bottom of page. Delete "s" from "preserves". (Should be: "... that inversions preserve angles")
- 69. *Page 319, line 8. Insert comma between P' and C'_2 .
- 70. Page 319, line 14. Change (*AB*, *PQ*) to [*AB*, *PQ*].
- 71. *Page 319, last line. Change *AP'* to *A'P'*.
- *Page 320, line 7. There is a left parenthesis missing from the last term in the last displayed equation of the proof.
- *Page 320, item 5 in summary. Add the add the assumption that *t* does not pass through 0.
- *Page 323, first line of Exercise 12.36. Add "is". (Should be: "Suppose T is")
- 75. *Page 323, first line of Exercise 12.42. Add comma after first *m*.
- 76. Page 324. Line 4: change exscribed to escribed.
- 77. Page 324, Exercise 12.52: Prove Theorem 12.7.16.
- 78. Page 324, Exercise 12.54(d): Add the word "Let" at the beginning of the sentence.
- 79. Page 324, Last line of footnote: Peaucellier
- *Page 350, third line of first full paragraph. Change "is is" to "it is."
- *Page 361, line 4. Delete "in." (Should be: "... produces a negatively curved surface.")
- 82. *Page 362, nine lines up from bottom of page. The word "as" is missing. (Should be: "... even though it is about as useless as anything could be....")
- 83. *Page 363, line 7. Insert "to." (Should be: "... allow you to verify....")
- *Page 370, line 26. Add the word "with." (Should be: "... in connection with the Ruler Postulate")
- *Page 374, line 4. Strike the word "of." (Should be: "... indirect way to measure the curvature....")
- *Page 392, 6 lines from bottom of page. Change "Arabic numbers" to "Arabic numerals".
- 87. *Page 394, line 10. Change "Postulated" to "postulated".
- *Page 415, fourth line of hint for Exercise 7.8. Line over AC missing. (Should be: "Show that m must intersect AC")
- *Page 416, second line of hint for Exercise 8.7. Replace B with B'. (Should be: "If X is on the same side of line OC as B'")
- 90. *Page 416, second line of hint for Exercise 9.6. The order of the vertices in the two similar triangles does not match. (Should be: "... prove that $\triangle AEC$ and $\triangle ADB$ are similar")
- 91. *Page 419, second line of the hint for Exercise 12.1. Strike the word "either." (Should be: "... there exist two points C and C' such that $\triangle ABC \cong \Delta f(A)f(B)P$ and $\triangle ABC' \cong \Delta f(A)f(B)P$")
- 92. *Page 419, second line of the hint for Exercise 12.10. Change "SAS" to "SSS." (Should be: "Use SSS to prove that....")
- 93. *Page 423, reference 48. Delete "The".
- 94. *Page 423, reference 60. Change "Campridge" to "Cambridge".



