Math 2280 HOMEWORK #9 Name:

Due 3/22/2024, 9:30 A.M.

Solve the following problems and staple your solutions to this cover sheet. (Computer outputs must be put in the appropriate place in the solution, not attached as an appendix. You may physically cut and paste the output in the problem or allow appropriate space in the printout to add your hand written work.)

1. Sec 4.7 #38
   Note: Must use $v_1' = \frac{-y_2\, g}{W(y_1, y_2)}$ and $v_2' = \frac{y_1\, g}{W(y_1, y_2)}$. Hint: $g(t) = \frac{t^3+1}{t^2}$.

2. Sec 4.7 #44
   Note: Must use $v_1' = \frac{-y_2\, g}{W(y_1, y_2)}$ and $v_2' = \frac{y_1\, g}{W(y_1, y_2)}$. Hints: $g(t) = t^{\frac{1}{2}}$. It turns out that $W(y_1,\, y_2) = t^{-1}$ and $y_p(t) = t^{\frac{1}{2}}$.

3. Find the general solution of $v'' + 2\,t^{-1}v' = 0$.
   Hint: Let $u = v'$. Then $v'' = u'$. See ERSE pages and your HW #4.

4. Find the general solution of $t\,v'' + (t-1)v' = 0$.
   Hint: Let $u = v'$. Then $v'' = u'$. See ERSE pages and your HW #4.

5. Sec 4.7 #46
   Notes: Must apply the method of reduction of order, not the formula in Theorem 8 of Sec 4.7. You can use the result of an earlier problem.

6. Sec 4.7 #47
   Notes: Must apply the method of reduction of order, not the formula in Theorem 8 of Sec 4.7. You can use the result of an earlier problem.

7. Find the general solution of $x^2\,y'' - 4x\,y' + 4y = \dfrac{1}{x^2}\;,\; x > 0\,.$
   Notes: Must make a guess for a solution to the corresponding homogeneous equation! Use it to find a second linearly independent solution of the homogeneous equation. (Must apply the method of reduction of order, not the formula in Theorem 8 of Sec 4.7.) Then, use both to find a particular solution of the original equation.

8. Sec 4.9 #2
   Hint: $A\cos(\omega t) + B\sin(\omega t) = C\sin(\omega t + \phi)$ where $C = \sqrt{A^2+B^2}$ and ϕ is the angle such that $\sin\phi = \frac{A}{C}$ and $\cos\phi = \frac{B}{C}$, or $\phi = \begin{cases} \tan^{-1}(\frac{A}{B}) & \text{, if } B > 0 \\ \pi + \tan^{-1}(\frac{A}{B}) & \text{, if } B < 0 \end{cases}$. See your first homework.

9. Sec 4.9 #10
   Hint: At the local maximum or minimum values of the distance, the velocity is zero (since the derivative is defined everywhere). The absolute maximum distance of the mass to right of its equilibrium point occurs at the first positive time at which the velocity is zero! **Why?**

10. Free points!