Ordinary Differential Equations - Math 2280 - Homework 1
Due at the start class on Friday, January 12
No late work will be accepted.

Name:_____

Your first homework is a review of necessary trigonometry and calculus skills for success in this course. You may use any reference to refresh your memory. You must write mathematically correct and will be graded on your writing. Very little partial credit will be given on this homework. **This homework should represent your best work!**

If you do not earn close to 100% on this homework due to any reason, such as, not knowing the material or not having time or care to refresh your memory**, you should drop this course.**

1. Find **all** solutions of $6\cos x - 8\sin x = 5$. Hint: Combine into one trigonometric function: $C\sin(x + \phi) = 5$.

2. Solve $w^3 = 1 - i$. Write your final answers in the form $a + b\,i$. Hint: Apply the nth root formula which is a consequence of the De Moivre's Theorem.

3. Find $f'(x)$ if $f(x) = e^{x^2}\left(\int_0^x e^{-t^2}\,dt\right)$. Hint: Use the product rule and the Fundamental Theorem of Calculus. Note: Any attempt to evaluate the integral is futile!

4. Find the value of the determinant $\begin{vmatrix} 0 & 1 & -2 & 3 \\ 2 & -1 & 2 & 3 \\ 0 & -3 & 4 & 5 \\ 0 & 2 & 3 & 4 \end{vmatrix}$.

5. Evaluate $\int x\sqrt{x+1}\,dx$ Hint: Use u-substitution.

6. Evaluate $\int t\, e^{-3t}\, dt$ Hint: Use integration by parts.

7. Evaluate $\int e^x \cos x\, dx$ Hint: Let $I = \int e^x \cos x\, dx$ and use integration by parts twice and solve for I.

8. Evaluate $\int \dfrac{3}{x(x^2+1)}\, dx$ Hint: Use partial fractions.

9. Find the Taylor series expansion of $f(x) = \ln x$ at $x_0 = 1$, in closed form. Hint: $\sum_{n=0}^{\infty} \frac{f^{(n)}(x_0)}{n!} (x - x_0)^n$.

10. Find the radius of convergence and interval of convergence of $\displaystyle\sum_{n=0}^{\infty} \frac{2^{-n}}{n+1} (x-1)^n$. Hint: Apply the ratio test and check for convergence at the endpoints.