Math 2280 HOMEWORK #5 Name:

Due 2/6/2026, 9:30 A.M.

Solve the following problems and staple your solutions to this cover sheet. (Computer outputs must be put in the appropriate place in the solution, not attached as an appendix. You may physically cut and paste the output in the problem or allow appropriate space in the printout to add your hand written work.)

1. Sec 3.2 #1
2. Sec 3.2 #4
3. Sec 3.2 #9 Must start with the equation 10: $\frac{dp}{dt} = kp$. Solve it and use the given information to find k and C. Hint: Use $t = 0$ for the year 1990. Note: We used a different notation in the class.
4. Sec 3.2 #13 Don't follow the hint in the book. Must start with the equation 14: $\frac{dp}{dt} = -Ap(p - p_1)$. Solve it and use the given information to find A, p_1 and C. Hint: Use $t = 0$ for the year 1990. Note: We used a different notation in the class.
5. Sec 3.2 #24 Must start with $\frac{dx}{dt} = -kx$. Solve it and use the given information to find k and C.
6. Sec 3.4 #2 Hint: The term $e^{-0.8t}$ in the equation for the time the object hits the ground can be ignored! Why? However, you may solve the equation for the time the object hits ground using Mathematica.
7. Sec 3.4 #3 Must start with appropriate ODE's and solve them. Hint: To solve the equation for the time the object hits the ground use Mathematica.

Next you are asked to solve Exercise 13 in Section 3.4 in two different ways.

8. Sec 3.4 #13 (Solution 1) Solution 1: With the up direction being the positive direction, $3\,\frac{dv}{dt} = -0.1v^2 - 29.43$, $v(0) = 500$. Solve for v and find the time the shell reaches its maximum height which is the time at which $v(t) = 0$. Then solve $\frac{dx}{dt} = v$, $x(0) = 0$ and find value of $x(t)$ at the time of maximum height.
9. Sec 3.4 #13 (Solution 2) Solution 2: You can also find the maximum height without finding the height function $x(t)$ as a function of t. Think of v as a function of x, $v = v(x)$. By the chain rule $\frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = \frac{dv}{dx}\,v$. For the independent variable x, the above IVP for the velocity can be written as $3\frac{dv}{dx}\,v = -0.1v^2 - 29.43$, with the initial condition at $x = 0$ being $v(0) = 500$. Solve for $v = v(x)$ and then find the maximum height by solving $v(x) = 0$ for x.
10. A body of mass $10\,kg$ is thrown downward with the initial velocity of $20\,m/sec$ in a medium offering resistsnce twice the square of the velocity. Find its velocity at any time t and determine its lmiting velocity, if it exists. Note: Use $g = 9.8$ in order to make the numbers for integration using partial fractions easier.
11. Free points!
12. Free points!

Note: See the Mathematica commands on the back.

Mathematica Commands

See your HW 2 for Mathematica commands. Below are just reminders of some useful commands for this homework.

To solve the equation $f(x) = g(x)$ numerically, do the following.

NSolve[$f(x)$==$g(x)$]

The NSolve may not work well. To apply the Newton's method to solve the equation $f(x) = g(x)$ with the starting x-value x_1, do the following.

FindRoot[$f(x)$==$g(x)$,{x,x_1}]