Math 2280 HOMEWORK #3 Name:

Due 1/23/2026, 9:30 A.M.

Solve the following problems and staple your solutions to this cover sheet. Computer work must be put in the appropriate place in the solution, not attached as an appendix. You may physically cut and paste the output in the problem or allow appropriate space in the printout to add your hand written work.

1. Sec 1.1 #8
2. Sec 1.2 #15 Note: Simplify both the Left Hand Side, LHS $= \frac{d\phi}{dx}$, and the Right Hand Side, RHS $= \frac{\phi(\phi-2)}{2}$, and show they are equal. You can draw several solutions by hand, but using Mathematica is recommended. **Be sure to label each solution.**
3. Sec 1.2 #20(a)
4. Sec 1.2 #25 Hint: Apply Theorem 1 of Section 1.2 with independent variable t in place of x and dependent variable x in place of y; $f(t, x) = -\frac{4t}{3x}$. Be sure to clearly state the open rectangle R and that the hypotheses of the theorem are satisfied.
5. Sec 1.3 #2 Note: Copy Fig 1.13, paste it in your homework, and use it for parts b and c. A solution cannot cross the x-axis since $\frac{4x}{y}$ is not defned on $y = 0$.
6. Sec 1.3 #14 Note: For $\frac{dy}{dx} = c$, at each point of the isoclines $\frac{x}{y} = c$, the tangent line to the solution has slope c. Draw the direction filed by hand!
7. Sec 1.3 #5 Note: Use Mathematica to draw the direction field.
8. Sec 2.2 #9 Hint: To integrate use the u substitution, $u = 1 + x$. Solve for y.
9. Sec 2.2 #11 Hint: To integrate use the trigonometric identity $\cos^2\theta = \frac{1}{2}(1 + \cos 2\theta)$. State the solution implicitly.
10. Solve the ODE $(x-2)\frac{dy}{dx} + \frac{1}{x+3}\,y = 0$. Assume $x > 2$. Hint: See the lecture notes.
11. Sec 2.2 #20 Hint: To integrate apply partial fractions and use $\int(y+1)\,dy = \frac{1}{2}(y+1)^2$. Find the explicit solution.
12. Sec 2.2 #21 Hint: To integrate use integration by parts.

Note: See the Mathematica commands on the back.

Mathematica Commands

See your HW 2 for the Mathematica commands. Below are just reminders of some useful commands for this homework.

To plot the function $y = f(x)$ for $a < x < b$, do the following.

`Plot[`$f(x)$`, {x, a, b}]`

To plot several functions $y = f_1(x)\,,\ y = f_1(x)\,,\cdots$ on the same coordinate system for $a < x < b$, do the following.

`Plot[{`$f_1(x),\ f_2(x),\ \cdots$`}, {x, a, b}]`

To plot a family of functions $y = f(x,\, c)$, for c values $c_1\,,\ c_2\,,\ \cdots$, on the same coordinate system for $a < x < b$, do the following.

`Plot[Table[f(x, c),{c,{`$c_1,\ c_2,\ \cdots$`}}],{x, a, b}]`

To plot a family of functions $y = f(x,\, c)$, for all integer c values from *lowc* to *highc*, on the same coordinate system for $a < x < b$, do the following.

`Plot[Table[f(x, c),{c,lowc,highc}],{x, a, b}]`

To plot the direction field for the ODE $y' = f(x,\, y)$ do the following.

`VectorPlot[{`$1,\ f(x,y)$`}, {`$x,\ a,\ b$`}, {`$y,\ c,\ d$`}]` (a, b and c, d are the lower and upper range of values of x and y, respectively.) The current version of Mathematica encodes the size of vectors in their colors and draws vectors of the same length, unless stated otherwise.

`VectorPlot[{`$1,\ f(x,y)$`}, {`$x,\ a,\ b$`}, {`$y,\ c,\ d$`}, VectorScaling->Automatic]` (This produces vectors proportional to their actual length, not all the same length.

`StreamPlot[{`$1,\ f(x,y)$`}, {`$x,\ a,\ b$`}, {`$y,\ c,\ d$`}]` (This commands embeds arrows onto flows!)