Math 2280 HOMEWORK #9 Name:

Due 3/21/2025, 8:30 A.M.

Solve the following problems and staple your solutions to this cover sheet. (Computer outputs must be put in the appropriate place in the solution, not attached as an appendix. You may physically cut and paste the output in the problem or allow appropriate space in the printout to add your hand written work.)

1. Sec 3.5 #1
   Hints: For an electrical circuit with resistance $R = 5\,ohms$, inductance $L = 0.05\,henry$, and voltage $E = 5\cos 120t\,volt$, the current I, in amperes, satisfies the equation $L\,\frac{dI}{dt} + R\,I = E$. Solve this first order ODE with the initial condition $I(0) = 1$. The solution $I(t)$ is the current. To find the voltage across the inductance E_L, in volts, calculate $E_L = L\,\frac{dI}{dt}$. Note: You can watch `https://faculty.weber.edu/aghoreishi/Math2280_S25/Sec3.5and5.7.mp4`.

2. Sec 5.7 #4
   Hints: For an electrical circuit with no resistance $R = 0\,ohm$, inductance $L = 2\,henry$, capacitance $C = 0.02\,farad$, and voltage $E = 30\sin 50t\,volt$, the charge q, in coulomb, satisfies the equation $L\,q''(t) + R\,q'(t) + \frac{1}{C}q(t) = E$. Solve this 2nd order linear nonhomogeneous ODE with the initial conditions $q(0) = 0$ and $q'(0) = 0$. The solution $q(t)$ is the charge. To find the current I, in amperes, calculate $I(t) = \frac{dq}{dt}$. Note: You can watch `https://faculty.weber.edu/aghoreishi/Math2280_S25/Sec3.5and5.7.mp4`.

3. Sec 4.9 #2
   Hint: $A\cos(\omega t) + B\sin(\omega t) = C\sin(\omega t + \phi)$ where $C = \sqrt{A^2 + B^2}$ and ϕ is the angle such that $\sin\phi = \frac{A}{C}$ and $\cos\phi = \frac{B}{C}$, or $\phi = \begin{cases} \tan^{-1}(\frac{A}{B}) & , \text{if } B > 0 \\ \pi + \tan^{-1}(\frac{A}{B}) & , \text{if } B < 0 \end{cases}$. See your first homework.

4. Sec 4.9 #4
   Hint: For each b value solve the the initial value problem and use Mathematica to graph it. We will get a simple harmonic motion when $b = 0$, other cases will be underdamped, critically damped and overdamped.

5. Sec 4.9 #6
   Hint: For each k value solve the the initial value problem and use Mathematica to graph it. We will get overdamped, critically damped and underdamped cases.

6. Sec 4.9 #10
   Hint: At the local maximum or minimum values of the distance, the velocity is zero (since the derivative is defined everywhere). The absolute maximum distance of the mass to right of its equilibrium point occurs at the first positive time at which the velocity is zero! **Why?**

For the next three problems consider the ODE $my'' + by' + ky = 0$ with constants m, b and k all positive.

7. Suppose $b^2 - 4mk > 0$. Prove all solutions $y(t)$ of this ODE will converge to zero as t increases, that is, $\lim_{t\to\infty} y(t) = 0$.
   Hint: See class notes. Need to show $r_1,\ r_2 < 0$.

Note: See the back.

8. Suppose $b^2 - 4mk = 0$. Prove all solutions $y(t)$ of this ODE will converge to zero as t increases, that is, $\lim_{t\to\infty} y(t) = 0$.
   Hint: See class notes. Need to show $r < 0$. Apply the L'Hospital's rule to one of the terms.

9. Suppose $b^2 - 4mk < 0$. Prove all solutions $y(t)$ of this ODE will converge to zero as t increases, that is, $\lim_{t\to\infty} y(t) = 0$.
   Hint: See class notes. Need to show $\alpha < 0$. Use the fact that $-e^{\alpha t} \le e^{\alpha t} \left\{ \begin{array}{c} \cos \beta t \\ \sin \beta t \end{array} \right\} \le e^{\alpha t}$ and apply the Squeeze Theorem.

10. Free points!

11. Free points!

12. Free points!

Mathematica Commands

See your HW 2 for Mathematica commands. Below are just reminders of some useful commands for this homework.

To plot the function $y = f(x)$ for $a < x < b$, do the following.

`Plot[`$f(x)$`, {x, a, b}]`

To plot several functions $y = f(x)$, $y = g(x)$, $\ldots$ on the same coordinate system for $a < x < b$, do the following.

`Plot[{f(x), g(x), ··· }, {x, a, b}]`

To plot a family of functions $y = f(x, c)$, for all integer c values from *lowc* to *highc*, on the same coordinate system for $a < x < b$, do the following.

`Plot[Table[f(x, c),{c,lowc,highc}],{x, a, b}]`

To solve the equation $f(x) = g(x)$ numerically, do the following.

`NSolve[`$f(x)$`==`$g(x)$`]`

The `NSolve` may not work well. To apply the Newton's method to solve the equation $f(x) = g(x)$ with the starting x-value x_1, do the following.

`FindRoot[`$f(x)$`==`$g(x)$`,{x,`x_1`}]`