Math 2280 HOMEWORK #10 Name:

Due 3/28/2025, 8:30 A.M.

Solve the following problems and staple your solutions to this cover sheet. (Computer outputs must be put in the appropriate place in the solution, not attached as an appendix. You may physically cut and paste the output in the problem or allow appropriate space in the printout to add your hand written work.)

1. Show that the function $f(t) = \sin bt$, $t \geq 0$, is piecewise continuous of exponential order zero. Use the definition of Laplace transform to find the Laplace transform of $f(t)$ for $s > 0$.
   Hints: Don't forget that piecewise continuity requires boundedness. State appropriate M and α values so that $|f(t)| \leq Me^{\alpha t}$. You may use $\int e^{ax} \sin(bx)\, dx = \frac{1}{a^2+b^2} e^{ax} [a \sin(bx) - b \cos(bx)] + C$. See class notes.

2. Sec 7.2 #8
   Hints: You have two options: (i) Show $f(t) = e^{-t} \sin bt$, $t \geq 0$, is a piecewise continuous of exponential order negative one and therefore its Laplace transform exists for $s > -1$, and then find it. Or, (ii) Use the definition of Laplace transform and show that the improper integral diverges for $s \leq -1$. Consider cases $s < -1$, $s = -1$, and $s > -1$. For either case, you may use $\int e^{ax} \sin(bx)\, dx = \frac{1}{a^2+b^2} e^{ax} [a \sin(bx) - b \cos(bx)] + C$. See class notes.

3. Sec 7.2 #10
   Hints: $\mathscr{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)\, dt = \int_0^1 e^{-st} f(t)\, dt + \int_1^\infty e^{-st} f(t)\, dt$. Consider the case $s = 0$ separately! See class notes.

4. Sec 7.2 #21

5. Sec 7.2 #26

6. Sec 7.3 #1

7. Sec 7.3 #21

8. Sec 7.4 #3

9. Sec 7.4 #25

10. Sec 7.5 #4

11. Sec 7.5 #10

12. Free points!

Note: See the Mathematica commands on the back.

Mathematica Commands

See your HW 2 for the Mathematica commands. Below are new commnads that can be helpful for this homework.

A piecewsie defined function can be inputted using the `Piecewise` command;

`f[x_]:=Piecewise[{ {`$f_1(x)$`, `$a_1 < x < b_1$`}, {`$f_2(x)$`, `$a_2 < x < b_2$`}, ⋯ } ]`

is the function $f(x) = \begin{cases} f_1(x)\,, & a_1 < x < b_1 \\ f_2(x)\,, & a_2 < x < b_2 \\ \vdots \end{cases}$.

The Mathematica notation for the unit step function $u(t-a)$ is `UnitStep[t-a]`.

The Mathematica notation for the Dirac delta function $\delta(t-a)$ is `DiracDelta[t-a]`.

The Mathematica notation for the Laplace transform is `LaplaceTransfrom[`$f(t)$`, t, s]` where $f(t)$ is the function and s is the independent variable of the Laplace transform $F(s)$.

The Mathematica notation for the inverse Laplace transform is `InverseLaplaceTransfrom[`$F(s)$`, s, t]` where $F(s)$ is the Laplace transform and t is the independent variable of the inverse $f(t)$.