Some Additional Review Problems from the Textbook

This is not an exhaustive list of all possible type of problems.
Answers and solutions to odd exercises are in the book and Student Solutions Manual, respectively.
(For more problems, see your class notes, examples in the book and homework problems.)

Section	Problems	Section	Problems
1.1	1-11 (odd)	6.1	3, 9, 15, 19
1.2	5, 7, 9, 15, 25	6.2	1, 5, 9, 13, 17
1.3	3, 5, 13	6.3	1, 5, 7, 31 (Not annihilator method)
2.2	1, 5, 11, 13, 27(a), 29	6.4	1, 5, 7
2.3	1, 3, 11, 21, 25(a)	Page 343	1(a), 2(c), 3, 5(a), 7(a), 9
2.4	1, 3, 7, 11, 19, 23	7.2	7, 11, 13, 17, 23, 29(a, b)
2.5	1, 3, 7, 9	7.3	3, 5, 7, 15, 21
Page 77	1, 3, 5, 7, 13, 19, 25, 29, 31, 35, 41	7.4	3, 5, 9, 21, 25
3.2	1, 3, 9, 13, 21	7.5	1, 3, 5, 7, 29
3.4	1, 5, 11, 13	7.6	5*, 7*, 11, 15, 29, 33, 35 * Use unit step functions
4.2	9, 17, 19, 27, 29	7.8	5, 9, 13, 17, 21
4.3	5, 17, 21, 23	Page 416	1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25
4.4	1, 3, 7, 13, 17, 27, 31	8.1	5, 13
4.5	1(a), 7, 9, 15, 21, 25, 27, 35	8.2	1, 9, 11, 19, 21, 23, 27, 29, 31
4.6	1, 7, 11, 15	8.3	1, 5, 7, 15, 17, 21, 27
4.7	1, 5, 37, 38, 45, 47	8.4	3, 9, 17, 21
4.9	1, 7, 9	8.5	3, 5, 9, 11, 13, 15
4.10	3, 9, 11	Page 491	1, 3, 5, 6
Page 233	1, 3, 5, 7, 9, 13, 15, 17, 19, 21, 23, 25, 29, 31, 33, 39		

Ordinary Differential Equations
Math 2280
Sample Final Exam
Sections 1.1-1.3, 2.1-2.5, 3.1, 3.2, 3.4, 4.1-4.7, 4.9
7.1-7.6, 7.8 & 8.1-8.5
Time Limit: 1 Hr and 50 Min 7 Pages

Name:__________________________

The point value of each problem is in the left-hand margin. You must show your work to receive any credit. Work neatly.

(12) 1. True or false.

( ) (a) The equation $5\,y' + 2x\,y^2 = 1$ is a linear differential equation.

( ) (b) The equation $y'' + x^2\,y' - 5\,y = 0$ is a homogeneous differential equation.

( ) (c) Functions $f(x) = 1 - x$ and $g(x) = x^2$ are linearly independent on $(-\infty,\,\infty)$.

( ) (d) The rational function $\frac{Q(x)}{P(x)}$ is analytic at all numbers x_0 such that $P(x_0) \neq 0$.

( ) (e) The equation $(y^2 + xy)\,dx + (2xy)\,dy = 0$ is an exact equation.

( ) (f) $y = e^{2x}$ is a solution of the ODE $y'' - y' + y = 3\,e^{2x}$.

(18) 2. Fill in the blanks.

(a) The particular solution of $y'' + 2y' + y = e^{-x}$ is of the form $y_p(x) =$.

(b) $\mathcal{L}\left\{u(t-\pi)\sin(t-\pi)\right\} =$.

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{(n-1)!} = \sum_{n=0}^{\infty}$ ______ .

(d) The minimum radius of convergence of a power series solution of $(x^2+1)y'' + xy' + (x+1)y = 0$ about $x = 1$ is .

(e) Two linearly independent solutions of $y'' - 4y' + 3y = 0$ are $y_1 =$ and $y_2 =$.

(f) $\mathscr{L}\left\{c_1\,f(t) + c_2\,g(t)\right\} = c_1\,\mathscr{L}\{\quad\} + c_2$.

(10) 3. Find the implicit solution of the IVP $2x(y+1)dx - y\,dy = 0$, $y(0) = -2$

(10) 4. Find the general solution of $\frac{dy}{dx} = y(2 + \sin x)$.

(15) 5. A body of mass $10\,kg$ is thrown downward with the initial velocity of $20\,m/sec$, in a medium offering resistance twice the square of the velocity. Determine its velocity at any time t and its limiting velocity (if it exists). Note: Clearly show the derivation of your differential equation!

(10) 6. A spring-mass-dashpot system has a mass of $1\,kg$ and its damping constant is $0.2\,\frac{N-Sec}{m}$. This mass can stretch the spring (without the dashpot) $9.8\,cm$. If the mass is pushed downward from its equilibrium position with a velocity of $1\,m/sec$, when will it attain its maximum displacement below its equilibrium?

(10) 7. Find the solution $y(x)$ of the IVP $x^2\, y'' - xy' + y = 0\,,\ y(1) = 1\,,\ y'(1) = 0$. Assume $x > 0$.

(10) 8. Find the general solution of $y'' + y = (\tan t)^2$.

(10) 9. Find the inverse Laplace transform of $G(s) = \frac{2s^2-4s+2}{(s^2-2s+5)(s+1)}$.

(10) 10. Find the solution of the IVP $y'' + 9y = 3\,\delta(t-\pi)$, $y(0) = 1$, $y'(0) = 0$.

(10) 11. Find the solution of the IVP $y'' + y = (t-1)u(t-1)$, $y(0) = y'(0) = 0$.

(10) 12. Find the first four nonzero terms in the Taylor series expansion of the solution of $(x^2 - 2x)y'' + 2y = 0$, $y(1) = -1$, $y'(1) = 3$ about $x_0 = 1$.

(15) 13. Find the general solution of $2y'' + xy' + y = 0$ using series solution method.

Brief Table of Laplace Transforms

$f(t) = \mathscr{L}^{-1}\{F(s)\}$	$F(s) = \mathscr{L}\{f(t)\}$
1	$\dfrac{1}{s}\,,\ s > 0$
e^{at}	$\dfrac{1}{s-a}\,,\ s > a$
$t^n\,,\ n = 1, 2, \cdots$	$\dfrac{n!}{s^{n+1}}\,,\ s > 0$
$\sin bt$	$\dfrac{b}{s^2+b^2}\,,\ s > 0$
$\cos bt$	$\dfrac{s}{s^2+b^2}\,,\ s > 0$
$e^{at}f(t)$	$F(s-a)$
$e^{at}\sin bt$	$\dfrac{b}{(s-a)^2+b^2}\,,\ s > a$
$e^{at}\cos bt$	$\dfrac{s-a}{(s-a)^2+b^2}\,,\ s > a$
$t^n e^{at}\,,\ n = 1, 2, \cdots$	$\dfrac{n!}{(s-a)^{n+1}}\,,\ s > a$
$f(at)\,,\ a > 0$	$\dfrac{1}{a}F\left(\dfrac{s}{a}\right)$
$u(t-a)\,,\ a \geq 0$	$\dfrac{e^{-as}}{s}\,,\ s > 0$
$u(t-a)f(t-a)\,,\ a \geq 0$	$e^{-as}F(s)$
$u(t-a)g(t)\,,\ a \geq 0$	$e^{-as}\mathscr{L}\{g(t+a)\}(s)$
$\delta(t-a)\,,\ a \geq 0$	e^{-as}
$\delta(t-a)f(t)\,,\ a \geq 0$	$e^{-as}f(a)$
$c\,f(t)$	$c\,F(s)$
$f(t)+g(t)$	$F(s)+G(s)\,,$ where $G(s) = \mathscr{L}\{g(t)\}$
$f'(t)$	$sF(s) - f(0)$
$f''(t)$	$s^2F(s) - sf(0) - f'(0)$
$f^{(n)}(t)$	$s^nF(s) - s^{n-1}f(0) - \cdots - sf^{(n-2)}(0) - f^{(n-1)}(0)$