Math 2280 HOMEWORK #9 Name:

Due 10/31/2025, 8:30 A.M.

Solve the following problems and staple your solutions to this cover sheet. (Computer outputs must be put in the appropriate place in the solution, not attached as an appendix. You may physically cut and paste the output in the problem or allow appropriate space in the printout to add your hand written work.)

1. Sec 3.5 #1
   Hints: For an electrical circuit with resistance $R = 5\,ohms$, inductance $L = 0.05\,henry$, and voltage $E = 5\cos(120t)\,volt$, the current I, in amperes, satisfies the equation $L\,\frac{dI}{dt} + R\,I = E$. Solve this first order ODE with the initial condition $I(0) = 1$. The solution $I(t)$ is the current. To find the voltage across inductance E_L, in volts, calculate $E_L = L\,\frac{dI}{dt}$. Note: You can watch `https://faculty.weber.edu/aghoreishi/Math2280_F25/Sec3.5and5.7.mp4`.

2. Sec 5.7 #4
   Hints: For an electrical circuit with no resistance $R = 0\,ohms$, inductance $L = 2\,henry$, capacitance $C = 0.02\,farad$, and voltage $E = 30\sin(50t)\,volt$, the charge q, in coulomb, satisfies the equation $L\,q''(t) + R\,q'(t) + \frac{1}{C}\,q(t) = E$. Solve this second order linear nonhomogeneous ODE with the initial conditions $q(0) = 0$ and $q'(0) = 0$. The solution $q(t)$ is the charge. To find the current I, in amperes, calculate $I(t) = \frac{dq}{dt}$. Note: You can watch `https://faculty.weber.edu/aghoreishi/Math2280_F25/Sec3.5and5.7.mp4`.

3. Show that the function $f(t) = \sin bt$, $t \geq 0$, is piecewise continuous of exponential order zero. Use the definition of Laplace transform to find the Laplace transform of $f(t)$ for $s > 0$.
   Hints: Don't forget that piecewise continuity requires boundedness. State appropriate M and α values so that $|f(t)| \leq Me^{\alpha t}$. You may use $\int e^{ax}\sin(bx)\,dx = \frac{1}{a^2+b^2}\,e^{ax}\,[a\sin(bx) - b\cos(bx)] + C$. See class notes.

4. Sec 7.2 #8
   Hints: You have two options: (i) Show $f(t) = e^{-t}\sin bt$, $t \geq 0$, is a piecewise continuous of exponential order negative one and therefore its Laplace transform exists for $s > -1$, and then find it. Or, (ii) Use the definition of Laplace transform and show that the improper integral diverges for $s \leq -1$. Consider cases $s < -1$, $s = -1$, and $s > -1$. For either case, you may use $\int e^{ax}\sin(bx)\,dx = \frac{1}{a^2+b^2}\,e^{ax}\,[a\sin(bx) - b\cos(bx)] + C$. See class notes.

5. Sec 7.2 #10
   Hints: $\mathscr{L}\{f(t)\} = \int_0^\infty e^{-st}\,f(t)\,dt = \int_0^1 e^{-st}\,f(t)\,dt + \int_1^\infty e^{-st}\,f(t)\,dt$. Consider the case $s = 0$ separately! See class notes.

6. Sec 7.2 #21

7. Sec 7.2 #26

8. Sec 7.3 #1

9. Sec 7.3 #21

10. Free points!

Note: Continued on the backside.

11. Free points!
12. Free points!

Mathematica Commands

See your HW 2 for the Mathematica commands. Below are some commands not in HW 2.

A piecewsie defined function can be inputted using the `Piecewise` command;

`f[x_]:=Piecewise[{ {`$f_1(x)$, $a_1 < x < b_1$`}, {`$f_2(x)$, $a_2 < x < b_2$`},` $\cdots$ `} ]`

is the function $f(x) = \begin{cases} f_1(x)\,, & a_1 < x < b_1 \\ f_2(x)\,, & a_2 < x < b_2 \\ \vdots & \end{cases}$.

The Mathematica notation for the unit step function $u(t-a)$ is `UnitStep[t-a]`.

The Mathematica notation for the Dirac delta function $\delta(t-a)$ is `DiracDelta[t-a]`.

The Mathematica notation for the Laplace transform is `LaplaceTransfrom[`$f(t)$`, t, s]` where $f(t)$ is the function and s is the independent variable of the Laplace transform $F(s)$.

The Mathematica notation for the inverse Laplace transform is `InverseLaplaceTransfrom[`$F(s)$`, s, t]` where $F(s)$ is the Laplace transform and t is the independent variable of the inverse $f(t)$.