Elementary Linear Algebra
Sample Final Exam - Part II - 4 pages
Math 2270

NAME: ______________________________

Time Limit: 1 Hour   Calculator Allowed: Scientific

The point value of each problem is in the left-hand margin. You must show your work to receive full credit for your answers, except on problem 1. Work neatly.

(10) 1. True or False.

(   ) (a) $AB = BA$ for all square matrices A and B of the same size.

(   ) (b) The square matrix A is invertible iff it is row equivalent to the identity matrix.

(   ) (c) The characteristic polynomial of the square matrix A in the variable λ is $\det(A - \lambda I)$.

(   ) (d) Every square matrix is diagonalizable.

(   ) (e) Any collection of n linearly independent vectors in $\Re^n$ form a basis for $\Re^n$.

(   ) (f) Let A be an $n \times n$ invertible matrix. Then the equation $A\vec{x} = \vec{b}$ has a unique solution for every vector $\vec{b}$ in $\Re^n$.

(   ) (g) If the equation $A\vec{x} = \vec{0}$ has 2 free variables, then the dimension of Nul A is 2.

(   ) (h) $\dim(\text{Col } A) + \dim(\text{Nul } A) =$ Number of rows of A.

(   ) (i) Suppose $\vec{u}$ and $\vec{v}$ are nonzero vectors and $\vec{u} \cdot \vec{v} = 0$. Then $\vec{u}$ and $\vec{v}$ are linearly dependent.

(   ) (j) It is possible for a 2×2 matrix with real-valued entries to have eigenvalues 3 and i.

(10) 2. State the definition of a subspace of vector space V.

(5) 3. Suppose the coordinates of $\vec{x}$ relative to the basis $\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ -5 \end{bmatrix}, \begin{bmatrix} 3 \\ -2 \\ 0 \end{bmatrix} \right\}$ is $[\vec{x}]_{\mathcal{B}} = \begin{bmatrix} 2 \\ -4 \\ 3 \end{bmatrix}$.

Find $\vec{x}$.

(10) 4. Let $A = \begin{bmatrix} 1 & 5 \\ -1 & -3 \end{bmatrix}$. Find eigenvalues of A. Show that A is similar to matrix of the form $\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$ by finding an appropriate invertible matrix P.

(10) 5. Find a basis for the subspace $H = \left\{ \begin{bmatrix} 3a + b + c \\ -b + 2c \\ -a - c \end{bmatrix} : a,\ b,\ c \in \Re \right\}$. Note: Basis vectors must be linearly independent!

(15) 6. Matrices $A = \begin{bmatrix} 2 & 4 & 1 & 6 & 1 \\ 3 & 6 & 2 & 7 & 0 \\ 2 & 4 & 0 & 10 & 4 \\ 3 & 6 & 2 & 7 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 4 & 1 & 6 & 1 \\ 0 & 0 & 1 & -4 & -3 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ are row equivalent. Find bases for the column space, row space and null space of A and state the dimension of each one.

(10) 7. Given $A = \begin{bmatrix} 3 & 2 & 3 \\ -6 & -1 & 5 \\ -6 & 2 & 1 \end{bmatrix}$. Find its LU factorization.

(10) 8. Find the inverse of the matrix $A = \begin{bmatrix} 1 & 2 & 0 \\ -1 & -3 & 2 \\ -1 & -2 & 1 \end{bmatrix}$

(10) 9. Given the vector space $V = \text{Span}\left\{ \begin{bmatrix} 5 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \\ 0 \end{bmatrix} \right\}$. Find an orthogonal basis for V.

(10) 10. Compare the values of determinants of the matrices $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ and $B = \begin{bmatrix} \frac{g}{3} & \frac{h}{3} & \frac{i}{3} \\ d - 2a & e - 2b & f - 2c \\ a & b & c \end{bmatrix}$.