Elementary Linear Algebra
Sample Final Exam - Part I - 3 pages
Math 2270

NAME: _____

No Time Limit - No Scratch Papers - Calculator Allowed: Scientific
Do not discuss this exam with anyone during the testing period.

The point value of each problem is in the left-hand margin. You must show your work to receive full credit for your answers. Work neatly.

(12) 1. Suppose square matrices A and B are similar. Show that A^2 and B^2 are also similar.

(14) 2. Let $\vec{u}$ and $\vec{v}$ be two vectors in $\Re^n$. Show that $\|\vec{u}+\vec{v}\|^2 + \|\vec{u}-\vec{v}\|^2 = 2\|\vec{u}\|^2 + 2\|\vec{v}\|^2$.

(12) 3. Prove that the mapping $T : \Re^3 \to \Re^2$ defined by $T(x,\, y,\, z) = (y + 2z,\, x - 2y)$ is linear.

(12) 4. Suppose the set $R = \{\vec{u},\, \vec{v}\,, \vec{w}\,\}$ is linearly independent. Show that the set $S = \{3\vec{u} + \vec{w}, \vec{v} - 2\vec{w},\, \vec{v} - \vec{u}\,\}$ is also linearly independent.

(12) 5. Show that $H = \left\{ \begin{bmatrix} a & -a \\ 2a & 0 \end{bmatrix} : a \in \Re \right\}$ is a subspace of $M_{2\times 2}$, the vector space of the collection of all 2×2 matrices with real-valued entries.

(14) 6. Let $\{\vec{u}_1, \cdots \vec{u}_n\}$ be an orthonormal basis for the vector space W. Show that if $\vec{y} = c_1\vec{u_1} + \cdots + c_2\vec{u}_2$, then $c_i = \vec{y} \cdot \vec{u}_i$ for $i = 1, \cdots, n$.

(12) 7. Suppose W is a subspace of $\Re^n$. Prove that $W^\perp$ is also a subspace of $\Re^n$.

(12) 8. Suppose A is an invertible matrix and λ is an eigenvalue of it. Show that $\frac{1}{\lambda}$ is an eigenvalue of A^{-1}.