Elementary Linear Algebra
Sample Exam II- 4 pages
Chapter 3, Sections 4.1-4.7, 5.1-5.3 & 5.5
Math 2270

NAME: ____________________

Time Limit: 50 Minutes   Calculator Allowed: Scientific

The point value of each problem is in the left-hand margin. You must show your work to receive full credit for your answers, except on problem 1. The set of real numbers is denoted by $\Re$. Work neatly.

(6) 1. True or False.

(   ) (a) Suppose A is a 3×5 matrix. If $\dim \text{Nul } A = 2$, then $\dim \text{row } A = 1$.

(   ) (b) The area of the parallelogram with the sides vectors $\vec{u}$ and $\vec{v}$ is the absolute value of the determinant of the matrix $A = [\vec{u} \quad \vec{v}]$.

(   ) (c) Let $\vec{x} \in \Re^2$ and $C = \begin{bmatrix} 1 & -\sqrt{3} \\ \sqrt{3} & 1 \end{bmatrix}$. Then $C\vec{x}$ rotates $\vec{x}$ by ______ degrees counterclockwise and then multiplies it by 2.

(7) 2. State the definition of a subspace of a vector space V.

(12) 3. Find the bases for the row space, the column space, and the null space of $A = \begin{bmatrix} 1 & -1 & -1 & 1 & 1 \\ -1 & 1 & 0 & -2 & 2 \\ 1 & -1 & -2 & 0 & 3 \\ 2 & -2 & -1 & 3 & 4 \end{bmatrix}$.

(10) 4. Find all eigenvalues of the matrix $A = \begin{bmatrix} 2 & -1 & 0 \\ 2 & 2 & 2 \\ -2 & 0 & 0 \end{bmatrix}$

(10) 5. The eigenvalues of the matrix $A = \begin{bmatrix} 11 & 16 \\ -8 & -13 \end{bmatrix}$ are $\lambda_1 = -5$ and $\lambda_1 = 3$. Diagonalize A.

(15) 6. Suppose T is a one-to-one linear transformation from vector space V onto vector space W. Suppose $\mathcal{B} = \{\vec{b}_1, \cdots, \vec{b}_n\}$ is a basis for V. Show that $\mathcal{D} = \{T(\vec{b}_1), \cdots, T(\vec{b}_n)\}$ is a basis for W.

(5) 7. Suppose $\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ -3 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix} \right\}$ is a basis for $\Re^3$ and let $\vec{x} = \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}$. Find $[\vec{x}\,]_{\mathcal{B}}$.

(5) 8. Compare the values of determinants of the matrices $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ and $B = \begin{bmatrix} g & h & i \\ 2d-a & 2e-b & 2f-c \\ a & b & c \end{bmatrix}$.

(10) 9. Find the determinant of the matrix $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ -2 & -4 & 5 & -9 \\ 3 & -4 & -1 & 6 \\ -1 & 3 & 5 & 7 \end{bmatrix}$ by first creating a matrix with the same determinant and containing several zeros.

(10) 10. Consider the subset $H = \{ax^2 + bx + c : a,\ b,\ c \in \Re \text{ and } a \neq 0\}$ of the vector space $I\!P_2 = \{ax^2 + bx + c : a,\ b,\ c \in \Re\}$. Either show H is a subspace of $I\!P_2$ or give an example to the contrary.

(10) 11. Prove that $\Re^2 = \text{Span}\left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \end{bmatrix} \right\}$.