

Elementary Linear Algebra
Sample Exam I - 4 pages
Sections 1.1-1.5, 1.7-1.9, 2.1-2.3 & 2.5
Math 2270

NAME: _____

Time Limit: 50 Minutes Calculator Allowed: Scientific

The point value of each problem is in the left-hand margin. You must show your work to receive full credit for your answers, except on problem 1. Work neatly.

(10) 1. True or False.

- () (a) If A is invertible, then the transformation $T(\vec{x}) = A\vec{x}$ is both 1-1 and onto.
- () (b) Suppose C is a singular matrix. If $AC = BC$, then $A = B$.
- () (c) If the coefficient matrix A is a 3×5 matrix with three pivots, then the system $A\vec{x} = \vec{b}$ is consistent.
- () (d) The zero vector is in the span of any collection of vectors.
- () (e) If the equation $A\vec{x} = \vec{b}$ is inconsistent for some vector \vec{b} , then the only solution of $A\vec{x} = \vec{0}$ is the trivial solution.

(5) 2. State the definition of a linearly independent set of vectors.

(7) 3. Let $A = \begin{bmatrix} 1 & 3 & -2 \\ 2 & 4 & -5 \\ -3 & -5 & 8 \end{bmatrix}$. Find the vector solution of the homogeneous equation $A\vec{x} = \vec{0}$. If one solution of $A\vec{x} = \vec{b}$ is $\vec{x}_p = \begin{bmatrix} 3 \\ -2 \\ 7 \end{bmatrix}$, find the general solution of $A\vec{x} = \vec{b}$.

(8) 4. Find the inverse of the matrix $A = \begin{bmatrix} 2 & 3 & -6 \\ -5 & -4 & 10 \\ 2 & 1 & -3 \end{bmatrix}$.

(8) 5. Is the vector $\vec{u} = \begin{bmatrix} 2 \\ -5 \\ 4 \end{bmatrix}$ in the span of vectors $\vec{v} = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}$, $\vec{w} = \begin{bmatrix} 0 \\ 1 \\ 4 \end{bmatrix}$ and $\vec{z} = \begin{bmatrix} 1 \\ 0 \\ 5 \end{bmatrix}$. If yes, write \vec{u} as a linear combination of \vec{v} , \vec{w} and \vec{z} .

- (10) 6. Consider the set of vectors $R = \{\vec{u}, \vec{v}, \vec{w}\}$ and $S = \{\vec{u}, \vec{v}, \vec{w}, \vec{z}\}$ with each vector in \mathbb{R}^5 . Show that if R is linearly dependent, then so is S . Prove that if S is linearly independent, then R is also linearly independent.

- (10) 7. Suppose the linear transformation $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ first rotates a point 30° counterclockwise about the origin and then reflects it about the horizontal axis. Find the standard matrix of T .

- (10) 8. For what h value(s) the system whose augmented matrix is $\begin{bmatrix} 2 & \frac{h}{3} - 1 & -8 \\ -3 & h^2 & 12 \end{bmatrix}$ has a unique solution? Explain!

(10) 9. Find the LU factorization of the matrix $A = \begin{bmatrix} 2 & 1 & -3 & 5 \\ -6 & 4 & 8 & -12 \\ 4 & -12 & -4 & 7 \end{bmatrix}$.

(10) 10. Consider the transformation $T: \mathfrak{R}^2 \rightarrow \mathfrak{R}^3$ defined by $T(x, y) = (x - 2y, 3x + y, -5x + 4y)$. Is T a one-to-one? Is T onto \mathfrak{R}^3 ? Explain!

(10) 11. Given matrices $A = \begin{bmatrix} 1 & 3 \\ 1 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 0 & -1 \\ -2 & 4 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} 0 & -2 \\ 1 & -1 \\ 2 & 0 \end{bmatrix}$. Find $AB + C^T$.