An Introduction To Mathematica
Algebra & Single Variable Calculus

1. The Mathematics computer lab is located in Bldg 4, Rm 505. All computers in this lab have access to the latest version of Mathematica. All other PC computer labs should also have access to this software. Edit your document (remove extras and errors, ensure the rest works correctly) and turn in your print-out as your 1st computer homework.

2. Here is how to get on Mathematica:

   Get on a computer. (Some machines may require a log in process. Instruction and help should be available in the labs.) Move the mouse pointer to the **Mathematica icon**, which looks like the above star like symbol, and rapidly **click twice** using the **left mouse button**. You may find the Mathematica icon on the desktop or under the button Start - Math & Statistics - Mathematica 5 .

3. Entering information is like using a word processor. You can also use the arrow and backspace keys for editing. Try typing some text and practice editing it. Mathematica displays all work in "cells". The brackets on the far right indicate the cell. You can create a new cell for your next input by clicking the mouse below the cell you have been working in. First a horizontal line will appear and then when you type a new cell will be created.

   Throughout this lab and future labs input the statements in `True Type` font exactly as is!

4. Mathematica will do arithmetic operations using the usual symbols +, −, ∗, /, ˆ . A blank space between two terms also indicates multiplication but using ∗ might facilitate later editing. To see how it works, enter the following calculation followed by the appropriate **Evaluation** (Execution) key command; the **Enter** button on the number pad (not ⟵┘ on the main keyboard) should work on all computers.

   `4*8` **Enter** (in the number pad, not the Return key)

   What you typed will now be given a label of *In[1]:=* (for input line 1). The answer will be in the next line labeled *Out[1]:=* (output line 1). Now use Mathematica to evaluate the following. Remember to push the **Enter** key after each part. Check the answers using a calculator. If the output is the same as the input, you probably have made an error in typing.

   (a) `3+6*5` (Does Mathematica follow order of operation rules?)

   (b) `274/3` Try this again with a decimal point after either number.

   (c) `((3.24*6.791)-14.7)/(4.5+82/3)`
   Mathematica will not accept brackets or braces as computational grouping symbols. Try $[(3.24*6.791) - 14.7]/(4.5 + 82/3)$. Do you get the same answer?

   (d) `2.6^2.99` The symbol `^` is the power sign.

   (e) `3 5` (Don't forget the space between 3 and 5.)

5. All built-in Mathematica commands or words begin with a capital letter and use "brackets" if a grouping symbol is appropriate. Type in and evaluate each of the following.

   (a) `Log[12.7]` This is the natural logarithm $\ln 12.7$. Notice that `Log` stands for natural logarithm and a base need not be stated!

   (b) `E^2.5` This is $e^{2.5}$.

   (c) `Log[10,72.8]` This is the common logarithm of 72.8 or $\log_{10} 72.8$. Notice that if a base different from e is needed, it must be the first number.

   (d) `Sqrt[64.0]` This is $\sqrt{64}$.

   (e) `Abs[-2.5]` This is $|-2.5|$.

   (f) `Sin[Pi/6]` This is $\sin(\pi/6)$. Trigonomtric functions are defined as a function of radians. If a degree unit is used, it must be indiacted as shown next.

   (g) `Cos[30 Degree]` This is $\cos(30°)$.

   (h) `ArcTan[1]` This is $\tan^{-1}(1)$ and is given in radian.

   (i) `I*I` I is $i = \sqrt{-1}$.

6. You can find a numerical approximation using the function `N` and `NumberForm`. Type in and evaluate each of the following.

   (a) `E` This symbol stands for the exact value of e and will only be replaced with its numerical value if asked.

   (b) `N[E]` This is the approximate value of e.

   (c) `Pi` This symbol stands for the exact value of π and will only be replaced with its numerical value if asked.

   (d) `N[Pi]` This is the approximate value of π.

   (e) `Pi//N` This is another way of asking for the numerical value of the π.

   The function `N` calculates the answer to 16 significant figures (not decimal places) but Mathematica only displays the default number of places. For viewing the answer to more than the default number of display places but **no more than 16** significant figures you must use the `NumberForm` command along with `N`. Try the following.

   (f) `NumberForm[N[E],10]`

   (g) `NumberForm[N[Pi], 16]`

   You can use the function `N` by itself for calculating and displaying the answer for **more than 17** significant figures. Try the following.

   (h) `N[Pi, 17]` Did you get one more digit than the last part?

   (i) `N[Sqrt[2], 50]` This gives the value of the irrational number $\sqrt{2}$ to 50 significant figures.

7. You can also input mathematics in a more standard form using palettes. A basic palette is usually present on the top right hand side of the screen. (If it is not, you can get it by clicking consecutively on the buttons Files, Palettes, and BasicInput. In some types of computers, you may need to click first on Notebook, then Palettes, and finally BasicInput.) You can get a form by clicking on it in the palette. Then you can fill it in by clicking on each location and typing. Use the palette to type in and evaluate each of the following.

   (a) $3^{5.2}$

   (b) $\sqrt{37.4}$

   (c) $\frac{125.3}{72}$

8. The % sign allows you to call back a previous output line. To call back the output from the previous line enter %, two lines up %%, 3 lines up %%%, etc., or better yet, identify the number of the line (say line 6) and enter %6. Evaluate the following.

   (a) `N[E^2]`

   (b) `Log[%]` Is the answer correct? Try it again by inputting `Log [%*]`, where * is replaced by the number of the input line of `N[E^ 2]`. (Do not put any space between % and the input line number *)

9. Mathematica can work symbolically to perform algebraic operations. Try these.

   (a) `Expand[(a+1)^5]`

   (b) `Simplify[3x^2-x-9+x^2+7x+5]`

   (c) `Factor[x^4 - 1]`

   (d) `Together[2/(x-3)-1/(x+2)]` This performs the operation $\frac{2}{x-3} - \frac{1}{x+2}$.

   (e) `Apart[5/((x-3)(x+2))]` This writes the fraction $\frac{5}{(x-3)(x+2)}$ as a sum of fractions. You did this using partial fraction procedure.

   (f) `Cancel[(x^2-2x-3)/(x^2-9)]`

10. Replacement. A single equal sign indicates replacements. Type in the following and then evaluate them by pushing the **Enter** key.

```
x = 5; y = 12; z = a + b;
```

Enter all of them in just one cell; you may enter each on a separate line or all on one line. If you type in more than one on the same line, be sure to separate them by semicolons. The final semicolon in a line suppresses the final output. If you have used all the semicolons, you should not get any output! Now evaluate the following.

   (a) `x` This should give the assigned value of x which is 5.

   (b) `x y^2` Remember to insert the space between x and y^2 to indicate a product.

   (c) `Expand[z^x]` This is $(a+b)^5$.

11. The assignments for x, y and z are permanent; this means that to reuse these variables for something else requires that they first be "cleared". Type in the following and evaluate it.

    (a) `Clear[x,y,z]`

    (b) `x` You should get x and not its previous value of 5.
    It is a good idea to clear each variable after its use, otherwise its reuse will give unintended results.

12. In Mathematica, an equation is denoted by using two equal signs. For example, the equation `3x+2==x-5` represents the equation $3x+2=x-5$. You can find exact solutions of many equations using the function `Solve`. Type in and evaluate each of the following. The solution is given as a replacement value for the variable ({$x \to$ `answer`}).

    (a) `Solve[3x-8==4]` If the output is an empty set, either this equation has no solution or you have forgotten to clear the value of x earlier. Check you answer.

    (b) `Solve[z^2+2z==-2]` Did you clear z earlier? What type of numbers are the solutions? When does a quadratic equation have two complex conjugate solutions?

    (c) `Solve[a t-2==3a, t]`
    The `t` after the comma indicates that the equation should be solved for t and not a. Simplify your answer: `Simplify[%]`.

13. The Solve function can also be used to solve systems of equations. To solve two or more equations simultaneously, enter the equations and the variables you want to solve for as lists. In Mathematica's syntax a list is any collection of terms separated by commas and grouped by braces. Input each part in one cell and evaluate it.

    (a) `Solve[{x-2y==4,x-1==5y}, {x,y}]` This solves the 2×2 linear system of equations: $x-2y=4$ and $x-1=5y$.

    (b) `Solve[{x+y^2==2, y==-x^2+2},{x,y}]` This solves the system of equations $x+y^2=2$ and $y=-x^2+2$.

14. It might not be possible or even desirable to find exact solutions to certain equations. You can find numerical approximation of solutions of many equations using the `NSolve` function. Type in and evaluate each of the following.

    (a) `NSolve[2x-3==x^2-3x-4]`

    (b) `NSolve[2x-3==x^2-3x-4, x]` Did you get the same answer as in part a?

    (c) `NSolve[{x^2+y^2==16, y==x^2-2x+2}, {x,y}]`

15. The equal sign for entering functions is `:=` (not just =). Its independent variable is indicated by an underline ([shift] [-]) after it. Both are placed in one bracket pair. Do not use capital letters in naming functions since it might get mixed up with Mathematica's reserved functions. Enter the following functions and then perform the indicated steps.

    `f[x_]:=x^2 - 5; g[x_]:=Log[x]/x` The functions defined are $f(x)=x^2-5$ and $g(x)=\frac{\ln x}{x}$ and the independent variable for both is x. Here, a semicolon at the end of the line has no effect!

    (a) `f[3]`

    (b) `g[f[3]]`

    (c) `Solve[f[x]==4]` This solves the equation $x^2-5=4$

    (d) `Clear[f,g]`

16. You can draw a function using the `Plot` command. `ParametricPlot` command is used to draw the graph of a parametric equation. The Mathematica syntax requires stating the independent variable(s) and the domain (minimum and maximum values of each independent variable). Enter the following.

    (a) `Plot[2Sin[x+Pi/4]-Cos[x], {x, -4Pi, 4Pi}]` This gives the graph of $y = 2\sin(x+\pi/4) - \cos x$ from $x = -4\pi$ to $x = 4\pi$. Redraw using the domain from -12 to 8.

    (b) `Plot[Log[Abs[x]],{x,-6,6}]` This gives the graph of $y = \ln|x|$ from $x = -6$ to $x = 6$.You might see an error message since for $x = 0$, $\ln|x|$ is undefined.

    (c) `ParametricPlot[{t Cos[t], Sin[t]}, {t, 0, 10}]` This is the graph of the parametric equation $x = t\cos t$ and $y = \sin t$.

    (d) To graph two or more functions on the same coordinate system, enter them as "lists" which means grouped in braces and separated by commas.

        `Plot[{Cos[x], x}, {x,-2Pi,2Pi}]` This is the graph of the two functions $y = \cos x$ and $y = x$ on the same coordinate system.

    (e) You can read the coordinates of the point on a graph by first clicking on the graph so a rectangular box appears around it, and next holding down the Ctrl key (the Alt key or equivalent on some machines) while moving the cursor over it. The coordinates will appear in the lower bar of your notebook.
        Click on the graph of part d. Hold down the Ctrl key (the Alt key or equivalent on some machines) and move the cursor over to the point of the intersection of the two graphs. What is the coordinates of this point? Your answer should be close to $(0.87, 0.66)$.

17. When the `NSolve` command fails, the command `FindRoot` can often give an accurate approximation of a solution. The `FindRoot` command uses a numerical method for approximating solutions which requires a starting point "close" to the actual solution. An appropriate graph can give a rough estimate of a solution for use as a starting point. Try the following.

    (a) `NSolve[Cos[x]==x^3, x]` The `NSolve` command fails to solve the equation $\cos x = x^3$.

        The x-coordinate of the point of intersection read above is close enough. This value was close to one. However, in this case there is no bad estimate.

    (b) `FindRoot[Cos[x]==x^3, {x, 1}]` The value 1 is the starting point of the numerical search for the solution in the `FindRoot` command. You should get `{x → 0.865474}`. Use a different starting point in place of 1 and evaluate it again. Do you get the same answer?

18. Mathematica has extra packages which contain additional functionalities. A package is inputted as `<<package`. Next, we will use a graphics package for plotting polar functions. Input the following.

    (a) ``<<Graphics`Graphics` `` The symbol ` is located on the same key as tilde, ~. The graphics package is now loaded.

    (b) `PolarPlot[Sin[2t], {t, 0, 2Pi}]` This is the plot of the polar function $r(\theta) = \sin 2\theta$, for θ's between 0 and 2π. (The keyboard doesn't have a key for θ so in its place we have used `t`. Of course you could use θ from the palette.)

    (c) If you have used the `PolarPlot` command before the loading package, then it will **not** work! You must remove this command using `Remove[PolarPlot]` and then loading the graphics package.

19. We can use Mathematica to evaluate one dimensional limits. Enter the following.

   (a) `Limit[Sin[x]/x, x->0, Direction->-1]` This evaluates the right limit $\lim_{x\to 0+} \frac{\sin x}{x}$.

   (b) `Limit[Sin[x]/x, x->0, Direction->+1]` This evaluates the left limit $\lim_{x\to 0-} \frac{\sin x}{x}$.

   (b) Since the left and right limit values are equal to 1, we have $\lim_{x\to 0} \frac{\sin x}{x} = 1$. Mathematica will give the same answer.
   `Limit[Sin[x]/x, x->0]` This evaluates limit $\lim_{x\to 0} \frac{\sin x}{x}$, assuming it exists.

   (c) Mathematica tends to find the right limit only and not check whether the left limit is the same value as the right limit. Therefore, if the limit does not exit, Mathematica will probably give the wrong answer.

   `Limit[Abs[x]/x, x->0]` The Mathematica's answer is one. But $\lim_{x\to 0} \frac{|x|}{x} \neq 1$ since the left and right limits are not equal, as is evident from the following. Here, Mathematica did not give the correct answer.
   `Limit[Abs[x]/x, x->0, Direction->+1]`

   (d) `Limit[Log[x]/Sqrt[x], x->Infinity]` This is $\lim_{x\to\infty} \frac{\ln x}{\sqrt{x}}$ which you learned to find using the l'Hospital's rule.

20. Mathematica can do symbolic differentiation. Input the following.

   (b) `D[x^2 Sin[x]-3x+1, x]` This gives the derivative of $y = x^2 \sin x - 3x + 1$ with respect to x.

   (b) You can also use the prime notation.
   `f[x_]:=x^3-2x^2+5;`
   `f'[x]` This is $f'(x)$.
   `Clear[f]`

   (c) `f[x_]:= x^3 ArcTan[x+1];`
   `f''[x]` This is the second derivative $f''(x)$ for $f(x) = x^3 \tan^{-1}(x+1)$.
   `D[f[x], {x, 3}]` This is the third derivative $f'''(x)$.
   `Clear[f]`

   (d) We can also find the derivative of a function defined implicitly. Let's try to find the derivative y' in terms of x and y if $x^2y^2 + x\sin y = 1$.

   `D[x^2 y[x]^2 + x Sin[y[x]]==1, x]` By `y[x]` we mean that y is a function of x.
   `Solve[% , y'[x]]` Here the last equation is solved for y'.

21. Mathematica can find many antiderivatives symbolically using the `Integrate` command. Mathematica can evaluate a definite integral by first finding the antiderivative of the integrand and applying the Fundamental Theorem of Calculus. However, if the antiderivative is not known or is difficult to find, then a definite integral can be evaluated numerically by the `NIntegrate` command. Input the following.

    (a) `Integrate [x^2+1, x]` This evaluates $\int (x^2+1)\,dx$.

    (b) `Integrate [x^2+1, {x, -1, 2}]` This evaluates $\int_{-1}^{2} (x^2+1)\,dx$.

    (c) `NIntegrate[Exp[-x^2], {x, 0, Infinity}]` This evaluates $\int_0^\infty e^{-x^2}\,dx$ numerically.

    (d) `Integrate [x^2 ArcSin[x], x]` This evaluates $\int x^2 \sin^{-1} x\,dx$. Mathematica can do many complicated integrals while we can do certain integrals more efficiently than Mathematica.

    (e) `Integrate [2x(x^2-1)^100, x]` Evaluate $\int 2x(x^2-1)^{100}\,dx$ by hand. Is your u-substitution method more efficient than the Mathematica's method?

22. The `Series` command can be used to find the power series expansion of a function about a given point up to a given desired power. Input the following.

    (a) `Series[E^x, {x,0,6}]` This is the Maclaurin series of $f(x) = e^x$ with all terms to the sixth power stated explicitly and $\mathtt{O[x]}^7$ stands for all higher order terms.

    (b) By adding the command `Normal` a series is truncated to a polynomial.
    `Normal[Series[Cos[x], {x, 0, 8}]]` This is the 8th-degree Maclaurin polynomial of $f(x) = \cos x$.

    (c) `Series[Log[x], {x,1,5}]` This is the Taylor series expansion of $f(x) = \ln x$ about $x = 1$ with all terms to the fifth power stated explicitly.

23. It is essential, both as a courtesy to future users, and to continued problem-free usage, to leave the computers as you find them. When you are done, close the software being used and/or log off properly.