Elementary Linear Algebra - Math 2270
Mathematica Commands

1. The Mathematics computer lab is located in Bldg 4, Rm 505. All computers in this lab have Mathematica 5 installed. Other computer labs should have access to this software. Edit your document (remove extras and errors, ensure the rest works correctly) and turn in your print-out as your 2nd computer homework.

2. We will use the package for vector calculus. Input the following. **Input the statements in `True Type` font exactly as is**!

   You may input the mathematics as below or in a more standard form using palettes. A basic palette is usually present on the top right hand side of the screen. (If it is not, you can get it by clicking consecutively on the buttons Files, Palettes, and BasicInput.

3. Braces, { }, signal a "list" to Mathematica. A list can be used to represent a set, an interval, a table of data, the parameters in a command, a vector, a matrix, etc. For a matrix, each row is inputted as a list in the order it appears and rows are separated by commas. If a row has only one entry, it is not necessary to use braces. Now try the following.

   (a) `u={1,-3}` This is the vector $\vec{u} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$.

   (b) `MatrixForm[u]` This command shows $\vec{u}$ in matrix form.

   (c) `v={{-2,5,-1}}` This is the row vector $\vec{v} = \begin{bmatrix} -2 & 5 & -1 \end{bmatrix}$.
   `MatrixForm[v]`

   (d) `m={{1,2},{3,4},{5,6}}` This is the matrix $m = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$.

   `MatrixForm[m]`

4. You can add or subtract matrices using $+$ and $-$ signs. The symbol "`.`" is the multiplication sign for matrices. Input the following.

   (a) `n={{3,2,1},{6,5,4}}` This is the matrix $n = \begin{bmatrix} 3 & 2 & 1 \\ 6 & 5 & 4 \end{bmatrix}$.

   `l={{3,-1},{-2,6},{-4,5}}` This is the matrix $l = \begin{bmatrix} 3 & -1 \\ -2 & 6 \\ -4 & 5 \end{bmatrix}$.

   (b) `m - 2*l` The result is the matrix $m - 2l$.

   (c) `n.m` This gives the product nm.
   `MatrixForm[n.m]`

5. Try the following.

   (a) `Dimensions[m]` This gives the size of the matrix m. The first number is the number of rows and the second is the number of columns.

   (b) `i=IdentityMatrix[3]` This is the 3×3 identity matrix i.
   `MatrixForm[i]`

   (c) `d=DiagonalMatrix[{2,2,-3}]` This is the 3×3 diagonal matrix d with the diagonal entries 2, 2 and -3.

   (d) `Transpose[m]` This gives the transpose of the matrix m.
   `MatrixForm[Transpose[m]]`

6. Here we will perform more matrix operations. Input the following.

   (a) `a={{2,3,-5},{-1,4,2},{5,7,2}}`

   (b) `Det[a]` This gives the determinant of a.

   (c) `Inverse[a]` This gives the inverse of the matrix a.
   `MatrixForm[Inverse[a]]`

   (d) `MatrixPower[a,3]` This gives the matrix a^3.

7. More matrix operations and solving linear systems of equations. Input the following.

   (a) `RowReduce[m]` This gives the reduced echelon form of m.

   (b) `RowReduce[a]` Since a is invertible, the reduced echelon form of a is the identity matrix.

   Now, consider the system $\begin{cases} 2x+3y-5z=12 \\ -x+4y+2z=-7 \\ 5x+7y+2z=4 \end{cases}$ which in the matrix form is $\begin{bmatrix} 2 & 3 & -5 \\ -1 & 4 & 2 \\ 5 & 7 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ -7 \\ 4 \end{bmatrix}$.

   (c) `Solve[{2x+3y-5z==12,-x+4y+2z==-7,5x+7y+2z==4},{x,y,z}]` By the `Solve` command, we can find the solution of this system.

   (d) `auga={{2,3,-5,12},{-1,4,2,-7},{5,7,2,4}}` This is the augmented matrix for the matrix equation.

   (e) `RowReduce[auga]` From the reduced echelon form of the augmented matrix, we can see the solution is $x = 319/159$, $y = -55/159$ and $z = -287/159$.

   (f) `b={12,-7,4}` This is the constant matrix on the right side.

   (g) `Inverse[a].b` Another way to find the solution is to calculate $a^{-1}b$. Did you get the answer as in part (d)?

   (h) `LinearSolve[a,b]` The `LinearSolve` command uses the LU factorization to solve the system. Did you get the answer as in part (d)?

8. Eigenvalues and eigenvectors. Input the following.

   (a) `c={{8,6,3},{-3,-1,-3},{-3,0,2}}`

   `Eigenvalues[c]` Here we get the eigenvalues of the new matrix $c = \begin{bmatrix} 8 & 6 & 3 \\ -3 & -1 & -3 \\ 3 & 0 & 2 \end{bmatrix}$.

   This matrix has one real and a pair of complex conjugate eigenvalues.

   (b) `Eigenvectors[c]` This gives the eigenvectors of c. Notice two of them are complex conjugates.

   (c) `Eigensystem[c]` This gives a list of eigenvalues followed by their corresponding eigenvectors.

9. Here is one package from many useful packages. Input the following.

   (a) ``<<LinearAlgebra`Orthogonalization` `` There are no blank spaces. The first two symbols are less than signs and the symbol ` is on the same key with ~ and is usually on the far left side next to the key for the number 1. This loads the package used for the Gram-Schmidt process.

   (b) `GramSchmidt[{{3,4,2}, {2,5,2}, {1,2,6}}]` This produces an orthonormal basis for $\Re^3$ from the basis $\mathcal{B} = \left\{ \begin{bmatrix} 3 \\ 4 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \\ 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 6 \end{bmatrix} \right\}$.

   To check that these vectors are indeed orthonormal, let's name the output vectors of this process. This way we do not have to type in each one.

   (c) `{u, v, w} = GramSchmidt[{{3,4,2}, {2,5,2}, {1,2,6}}]` The three vectors are labeled `u`, `v` and `w`.

   To check their length and orthogonality, we need to find their dot (or inner) products. The symbol "`.`" is also used for the dot product.

   (d) `{u.v, u.w, v.w, u.u, v.v, w.w}` This gives the dot products $\vec{u} \cdot \vec{v}$, $\vec{u} \cdot \vec{w}$, $\vec{v} \cdot \vec{w}$, $\vec{u} \cdot \vec{u}$, $\vec{v} \cdot \vec{v}$, $\vec{w} \cdot \vec{w}$. Are these vectors orthonormal?

10. It is essential, both as a courtesy to future users, and to continued problem-free usage, to leave the computers as you find them. When you are done, close the software being used and/or log off properly.