Calculus III
Sample Final Exam - 6 pages
Math 2210

NAME: ______________________________

TIME LIMIT **2** HOURS

The point value of each problem is in the left-hand margin. You must show your work to receive full credit for your answers, except on problem 1. Work neatly.

(10) 1. Fill in the blanks.

(a) The level curves of $z = x^2 + y^2$ are ______________ .

(b) A fluid with velocity vector field $\vec{V}$ is said to be incompressible if ______________ .

(c) According to the Stokes' Theorem $\int_C \vec{F} \cdot d\vec{r} =$ ______________ where C is the boundary of the surface S with positive orientation with respect to the orientation of S.

(d) The distance between points $(1,\, 4,\, -8)$ and $(3,\, 2,\, -9)$ is ______________ units.

(e) Suppose $f(x,\, y)$ is differentiable. At any point $(a,\, b)$, the direction at which $z = f(x,\, y)$ increases most rapidly is ______________ .

(10) 2. Find the absolute maximum and minimum values of $f(x,\, y) = 2 + 2x + 2y - x^2 - y^2$ on the triangular plate in the first quadrant bounded by the lines $x = 0$, $y = 0$, $y = 9 - x$.

(10) 3. Find the value of $\displaystyle\lim_{(x,\,y)\to(0,\,0)} \frac{x^2 - 2xy}{x^2 + y^2}$ or show the limit does not exist.

(10) 4. Evaluate $\displaystyle\int_0^{\pi}\int_x^{\pi} \frac{\sin y}{y}\, dy\, dx$.

(5) 5. Write $\displaystyle\iiint_E f(x,\, y,\, z)\, dV$ as an triple iterated integral if E is the solid tetrahedron with vertices $(0,\, 0,\, 0)$, $(1,\, 1,\, 0)$, $(0,\, 1,\, 0)$ and $(0,\, 1,\, 1)$.

(10) 6. Suppose the components of the vector field $\vec{F}(x, y, z) = F_1(x,\ y,\ z)\,\vec{i} + F_2(x,\ y,\ z)\,\vec{j} + F_3(x,\ y,\ z)\,\vec{k}$ have continuous second order partial derivatives. Show that $\operatorname{div}\operatorname{curl}\vec{F} = 0$.

(10) 7. Find a spherical coordinate equation of the surface $x^2+y^2+(z-1)^2 = 1$ given in cartesian coordinates.

(10) 8. Evaluate $\displaystyle\int_{-1}^{1}\int_{0}^{\sqrt{1-x^2}}\int_{0}^{x}(x^2+y^2)\,dz\,dy\,dx$.

(10) 9. Use the divergence theorem to find the flux of $\vec{F} = x^2\,\vec{i} + y\,\vec{i} - z\,\vec{k}$ through the boundary surface of the solid in the first octant bounded above by $z = -2x - 3y + 6$, and oriented outward.

(10) 10. Use the Green's theorem to evaluate the line integral $\int_C \vec{F} \cdot d\vec{r}$, where $\vec{F} = y^{\frac{3}{2}}\,\vec{i} + x\,\vec{j}$ and C is the boundary of the region bounded by $y = x^2$, $y = 0$, $x = 0$, and $x = 2$, with counterclockwise orientation.

(10) 11. Evaluate $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F} = -y\vec{\imath} + x\vec{\jmath}$ and C is the parabola $y = x^2 - 1$ from $(-1,\ 0)$ to $(2,\ 3)$.

(10) 12. Given $f(x, y) = x^2y + y^2 \cos x + 1$, find $\dfrac{\partial f}{\partial x}$, $\dfrac{\partial^2 f}{\partial y \partial x}$, and $f_{xy\,x}$.

(10) 13. Find the directional derivative of $f(x,\ y) = x^2 - 2y^2$ in the direction of the vector $\vec{v} = \vec{\imath} - 2\vec{\jmath}$ at the point $(1,\ -1)$.

(10) 14. Evaluate $\displaystyle\int_0^4 \int_{\frac{y}{2}}^{\frac{y}{2}+1} \frac{2x-y}{2}\, dx\, dy$ by applying the transformation $u = \frac{2x-y}{2}$ and $v = \frac{y}{2}$.

(10) 15. Find the distance of the point $(2, -3, 5)$ from the plane $x - 2y + z = 1$.

(5) 16. Find the equation of the tangent plane to the surface $x^2 + y^2 + z - 9 = 0$ at the point $(1, 2, 4)$.