

Calculus III
Sample Exam II - 4 pages
Chapter 15 & Sections 16.1-16.4
Math 2210

NAME: _____

TIME LIMIT 50 MINUTES

The point value of each problem is in the left-hand margin. You must show your work to receive full credit for your answers, except on problem 1. Work neatly.

(10) 1. Fill in the blanks.

(a) The graph of the domain of the function $f(x, y) = \sqrt{1 - x^2 - y^2}$ is _____ .

(b) The critical points of the function $z = f(x, y)$ are points at which _____ .

(c) $\int_a^b \int_c^d f(x, y) dy dx = \int \int f(x, y) dx dy$.

(d) If $f_x(x_0, y_0) = f_y(x_0, y_0) = 0$, then f has a local minimum at (x_0, y_0) if _____ .

(e) Suppose $f(x, y)$ is differentiable. At any point (a, b) , the direction at which $z = f(x, y)$ increases most rapidly is _____ .

(10) 2. Find the directional derivative of $f(x, y) = x^2 - 2y^2$ in the direction of the vector $\vec{v} = \vec{i} - 2\vec{j}$ at the point $(1, -1)$.

(10) 3. Given $f(x, y) = x^2y + y^2 \cos x + 1$, find $\frac{\partial f}{\partial x}$, $\frac{\partial^2 f}{\partial y \partial x}$, and f_{xyx} .

(10) 4. Either evaluate $\lim_{(x,y) \rightarrow (1,-1)} \frac{(x-1)^2}{x^2 - 2x - y}$ or show it does not exist.

(10) 5. (a) Find the linearization of $f(x, y) = \sqrt{x + e^{2y}}$ at the point $(3, 0)$. (b) Use the linearization to approximate $f(2.8, 0.1)$.

(10) 6. Find the distance of the point $(2, -3, 5)$ from the plane $x - 2y + z = 1$.

(10) 7. Find all maxima, minima, and saddle points of the function $f(x, y) = x^3 + y^3 + 3x^2 - 3y^2$.

(10) 8. By changing the order of integration show that

$$\int_0^a \left(\int_0^y f(x) dx \right) dy = \int_0^a (a-x)f(x) dx$$

where f is a continuous function. Be sure to clearly show all steps!

(10) 9. Find the volume of the solid bounded by the two elliptic paraboloids $z = 12 - x^2 - y^2$ and $z = 2x^2 + 2y^2$.

(10) 10. Find three real numbers whose sum is 9 and the sum of their squares is as small as possible. Note: You must show that your answer is the one with the least sum of squares.