Calculus III
Sample Exam I - 4 pages
Chapters 13 & 14
Math 2210

NAME: _____

TIME LIMIT 50 MINUTES

The point value of each problem is in the left-hand margin. You must show your work to receive full credit for your answers, except on problem 1. Work neatly.

(10) 1. Fill in the blanks.

(a) For a vector $\vec{v}$ in space $\vec{v} \times \vec{v} =$ _____ .

(b) Two nonzero vectors $\vec{v}$ and $\vec{w}$ are orthogonal iff $\vec{v} \cdot \vec{w} =$ _____ .

(c) A vector perpendicular to the plane $2x - 3y + 5z + 1 = 0$ is $\vec{N} =$ _____ .

(d) If the velocity vector of an object is given by $\vec{r}(t) = 3t^2\,\vec{i} - 2\,\vec{j} + e^t\,\vec{k}$, then its velocity function is $\vec{v}(t) =$ _____ .

(e) The distance between points $(4, 3, -8)$ and $(-1, 5, 4)$ is _____ units.

(10) 2. Discuss the intersection of the graph of the surface $z = 4 - x^2 - y^2$ with coordinate planes and use them to graph this surface.

(10) 3. Find an equation of the plane through points $A = (-1,\ 2,\ -3)$, $B = (-2,\ 3,\ 1)$ and $C = (3,\ -1,\ 2)$.

(10) 4. Find an equation of the tangent line to the space curve $\vec{r}(t) = \langle \cos t,\ e^t,\ t\,e^t \rangle$ at the point $(1,\ 1,\ 0)$.

(10) 5. Find the length of the smooth curve $\vec{r}(t) = \langle e^t,\ \sqrt{2}\,t,\ e^{-t} \rangle$ for $0 \le t \le 1$.

(10) 6. A projectile is fired with muzzle speed $160\, ft/sec$ and angle of elevation 30° from the ground level. How far away will it hit the ground? Assume air resistance is negligible. Note: $g = 32\frac{ft}{sec^2}$.

(10) 7. Show that if a particle moves with constant speed, then its velocity and acceleration vectors are orthogonal.

(10) 8. Find an equation of the line through the point $P = (2,\ 5,\ 1)$ which is parallel to the line of intersection of the planes $2x + 3y - z = 1$ and $-5x + y + 3z = 1$.

(10) 9. Perform the following conversion of coordinate for given points and equations. Write an equivalent cartesian, cylinderical, or spherical coordinates (equation) of he given points (equation). State the formulas and show you work.

(a) $P = (-3\sqrt{3},\, -3,\, 2)$ given in rectangular coordinates to cylinderical coordinates.

(b) $Q = (4,\, \frac{\pi}{3},\, \frac{3\pi}{4})$ given in spherical coordinates to rectangular coordinates.

(c) $z = \sqrt{x^2 + y^2}$ given in rectangular coordinates to spherical coordinates.

(10) 10. Find the unit tangent vector and the curvature of the space curve given by the vector function $\vec{r}(t) = \sin t\, \vec{i} - \cos t\, \vec{j} - t\, \vec{k}$.