Calculus I

Math 1210

Homework I

Due: Friday, August 29, at the start of the class

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Your first homework is a review of necessary algebra and trigonometry skills for success in this course. You may use any reference to refresh your memory. You must write mathematically correct and will be graded on your writing. Very little partial credit will be given on this homework. **This homework should represent your best work!**

Students who do not earn close to 100% on this homework tend to do very poorly in this class. Please take the necessary time to review the prerequisite material.

1.   Simplify $\dfrac{(16xy)^{\frac{1}{4}} \left(-8x^2 z^3\right)^{-\frac{1}{3}}}{\left(25^{-1} x^2 yz\right)^{-\frac{1}{2}}}$.   Hint: Apply exponent rules and combine like terms. In the final answer, all exponents must be positive.

2.   Simplify $\dfrac{5 + \sqrt{3}}{3 - \sqrt{2}} + \dfrac{4 - \sqrt{2}}{3 + 2\sqrt{2}}$.   Hint: Rationalize each denominator, combine the two fractions and combine like terms.

3.   Prove the identity $\dfrac{\sin\theta}{1+\cos\theta} + \dfrac{1+\cos\theta}{\sin\theta} = 2\csc\theta$.   Hint: Start from the left-hand side and simplify appropriately until you get the right-hand side.

4.   Prove the identity $\dfrac{\sin^3\theta + \cos^3\theta}{1 - 2\cos^2\theta} = \dfrac{\sec\theta - \sin\theta}{\tan\theta - 1}$.   Hint: Start from the left-hand side and simplify appropriately until you get the right-hand side. Factor the numerator using the sum of two cubes identity.

5.   Solve $|2 - 3x| - 5 < 0$.   Hint: $|u| < p \implies -p < u < p$.

6. Solve $\dfrac{3y-1}{2y+1} - \dfrac{y}{y+1} = \dfrac{1}{3}$.   Hint: Multiply both sides by the LCD of all denominators. Note: Be sure to check the potential solutions.

7. Find **all** solutions of $3\cos\theta + 3 = 2\sin^2\theta$ Hint: Rewrite using one trigonometric function and solve the resulting quadratic type equation. Note: Be sure to list all solutions.

8. Find **all** solutions of $\cos(2\theta) + 3 = 5\cos\theta$ Hint: Rewrite using one trigonometric function and solve the resulting quadratic type equation. Note: Be sure to list all solutions.

9. Find the sign of $f(x) = (1 - x^2)^{\frac{1}{3}} - 2x^2(1 - x^2)^{-\frac{2}{3}}$, (intervals on which f is positive or negative).   Hint: Simplify and find the x values at which f is not defined or is zero. Place these numbers on a real-number line and check the sign of f in the intervals these numbers divide the real number into.

10. Draw the graph of $f(x) = \dfrac{x^2}{x^2 - x - 2}$.   Hint: Find the domain, intercepts, asymptotes, and use them along with several appropriate points to draw its graph.