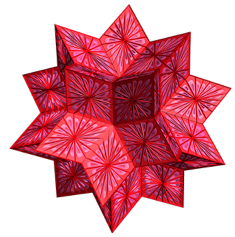
Mathematics Computer Laboratory - Math 1200 - Version 14

Lab 2 - Mathematica Basics Part II©



1. You may access the required software, called Mathematica, in the following three ways. 1. Get your own free copy by filling out the form at https://www.weber.edu/software/mathematica_request.html . 2. Any campus computer lab, including Tracy Hall Computer Lab, TY 126, and Elizabeth Hall Computer Lab, EH 213. 3. Virtual Lab: See http://www.weber.edu/virtuallab .

2. Here is how to get on Mathematica:

   Log in on a computer. Move the mouse pointer to the **Mathematica icon**, which looks like the above star like symbol, and rapidly **click twice** using the **left mouse button**. You may find the mathematica icon on the desktop, under the button Start - Programs - Wolfram or Mathematica, or by doing a search for Mathematica. Then click on NewDocument , or on New Notebook, if a notebook doesn't automatically open. You may also open a previous documnet by clicking on `Open...`. If you need help, ask the lab aide.

3. Recall that the **Evaluation** (Execution) key command is the **Enter** key on the **number pad**. (You may also use Shift ⟵┘.) Also remember that you should input the statements in `True Type` font **exactly as is**!

   Mathematica has three symbols for equality; let's try each one in the next three parts.

4. Replacement. A single equal sign indicates replacements. Type in the following and then evaluate them by pushing the **Enter** key.

   ```
   x = 5; y = 12; z = a + b;
   ```

   Enter all of them in just one cell; you may enter each on a separate line or all on one line. If you type in more than one on the same line, be sure to separate them by semicolons. The final semicolon in a line suppresses the final output. If you have used all the semicolons, you should not get any output! Now evaluate the following.

   (a) `x` (This should give the assigned value of x which is 5 .)
   (b) `z` (This should give the expression that z stands for: $a + b$.)
   (c) `x y^2` (Remember to insert the space between x and y to indicate a product.)
   (d) `Expand[z^x]` (This is $(a + b)^5$.)

5. The assignments for x , y and z are permanent; this means that to reuse these variables for something else requires that they first be "cleared". Type in the following and evaluate it.

   (a) `Clear[x,y,z]`
   (b) `x` (You should get x and not its previous value of 5 .)
   It is a good idea to clear each variable after its use, otherwise its reuse will give unintended results.

6. Input each part in one cell and evaluate it.
   (a) `f=x^2+3x+2; g=(2x+3)(5x-2);h=2/(x-3)-1/(x-1);k=(x+5)/((x+2)(x+3));` (As before, you may enter them on one or more lines.)
   (b) `f` (If the output is a number, you have forgotten to clear the variable x in the last step.)
   (c) `Factor[f]`
   (d) `Expand[g]`
   (e) `Together[h]`
   (f) `Apart[k]`
   (g) `Clear[f,g,h,k]`

7. In Mathematica, an equation is denoted by using two equal signs. For example, the equation `3x+2==x-5` represents the equation $3x+2=x-5$. You can find exact solutions of many equations using the command `Solve`. Type in and evaluate each of the following. The solution is given as a replacement value for the variable ($\{x \to$ `answer`$\}$).
   (a) `Solve[3x-8==4]` (If the output is an empty set, either this equation has no solution or you have forgotten to clear the value of x earlier. Check you answer.)
   (b) `Solve[z^2+2z==-2]` (Did you clear z earlier? What type of numbers are the solutions? When does a quadratic equation have two complex conjugate solutions?)
   (c) `Solve[a t-2==3a, t]`
   The t after the comma indicates that the equation should be solved for t and not a. Simplify your answer; (`Simplify[%]`).

8. Input each part in one cell and evaluate it.
   (a) `f = x^2-3x-4; Solve[f==0]` (This solves the equation $f=0$ where $f=x^2-3x-4$. Check your answers.)
   (b) `Clear[f]`
   (c) `g = 1/(x-2); h = 4/(x+1);`
   (d) `Solve[g==h]` (This solves the equation $\frac{1}{x-2}=\frac{4}{x+1}$.)
   (e) `Clear[g,h]`

9. The Solve command can also be used to solve systems of equations. Input each part in one cell and evaluate it.
   (a) `Solve[{x-2y==4,x-1==5y}, {x,y}]`
   This is the 2×2 linear system of equations: $x-2y=4$ and $x-1=5y$. To solve two or more equations simultaneously, enter the equations and the variables you want to solve for as lists. In Mathematica's syntax a list is any collection of terms separated by commas and grouped by braces.
   (b) `Solve[{3x-4y==4,x+2y==7}, {x,y}]`
   (c) `f = 2x+y; g = 5x+2y;`
   (d) `Solve[{f==2, g==3},{x,y}]` (This solves the system of equations $2x+y=2$ and $5x+2y=3$.)
   (f) `Clear[f,g]`

10. It might not be possible or even desirable to find exact solutions to certain equations. You can find numerical approximation of solutions of most equations using the `NSolve` function. Type in and evaluate each of the following.

    (a) `NSolve[2x-3==x^2-3x-4]`

    (b) `NSolve[2x-3==x^2-3x-4, x]` (Did you get the same answer as in part a?)

    (c) `NSolve[{x^2+y^2==16, y==x^2-2x+2}, {x,y}]`

11. The equal sign for entering functions is `:=` (not just =). Its independent variable is indicated by an underline ($\boxed{\text{shift}}$ $\boxed{\_}$) after it. Both are placed in one bracket pair. Do not use capital letters in naming functions since it might get mixed up with Mathematica's reserved functions. Enter the following functions and then perform the indicated steps.

    `f[x_]:=x^2 - 5; g[x_]:=Log[x]/x` (The functions defined are $f(x) = x^2 - 5$ and $g(x) = \frac{\ln x}{x}$ and the independent variable for both is x. Here, a semicolon at the end of the line has no effect!)

    (a) `f[3]`

    (b) `g[f[x]]`

    (c) `g[f[3]]`

    (d) Obtain a numerical approximation for part c. (Type in `N[g[f[3]]]` or `N[%]` line # of part (c)] .)

    (e) `Solve[f[x]==4]` This solves the equation $x^2 - 5 = 4$.

    (f) `Clear[f,g]`

12. Input each part in one cell and evaluate it.

    (a) `f[x_]:=1/(x-1); g[x_]:=(x+1)/x`

    (b) `f[g[x]];` Simplify this expression; (`Simplify[%]`).

    (c) `g[f[x]]` Simplify this expression.
    What can you deduce about these two functions from parts b & c?

    (d) `Clear[f,g]`

13. You can draw a function using the `Plot` command. The Mathematica syntax requires stating the independent variable(s) and the domain (minimum and maximum values of each independent variable). Enter the following.

    (a) `Plot[x^2-3x+4,{x,-10,10}]` (This gives the graph of $y = x^2 - 3x + 4$ from $x = -10$ to $x = 10$.)
    Redraw using the domain from -12 to 8 and then again from -0.1 to 1.2.

    (b) `Plot[Log[Abs[x]],{x,-6,6}]` (This gives the graph of $y = \ln|x|$ from $x = -6$ to $x = 6$.You might see an error message since for $x = 0$, $\ln|x|$ is undefined.)

    (c) `y = x^2-3x-10; Plot[y,{x,-5,8}]` (This gives the graph of $y = x^2 - 3x - 10$ from $x = -5$ to $x = 8$.)

    (d) `Clear[y]`

14. You can read the coordinates of a point on a graph by first right clicking on the graph and choosing Drawing Tools (or Get Coordinates) in the window that opens up. (In earlier versions of Mathematica, click on the graph, hold down the $\boxed{\text{Ctrl}}$ key and then push the $\boxed{\text{D}}$ key ($\boxed{\text{T}}$ for Mac OS).) In the new window, click on the symbol that looks like a cross hairs $\boxed{-\!|\!-}$ and move the cursor over the graph. The coordinates will appear on the screen. Try the following.

    (a) `f[x_]:=x+E^(-x); Plot[f[x], {x,-4,6}]` (This gives the graph of the function $f(x) = x+e^{-x}$ from $x = -4$ to $x = 6$.)

    (b) Click on the graph of part a. Hold down the $\boxed{\text{Ctrl}}$ key and then push the $\boxed{\text{d}}$ key. In the new window, click on the symbol that looks like a cross hairs $\boxed{-\!|\!-}$. Now, move the cursor over to a point on the graph with x-coordinate equal to 1. What is the y-coordinate of this point? (1, ?)

    (c) Check your answer in part b by finding the numerical value of $f(x)$ at $x = 1$; `f[1]`.

    (d) `Clear[f]`

15. You can plot two or more functions on the same coordinate system. Input each part in one cell and evaluate it.

    (a) `f[x_]:=-x^2+4x+5; g[x_]:=E^(x)`

    (b) `Plot[{f[x], g[x]}, {x,-2,4}]`
    To graph two or more functions on the same coordinate system, enter them as “lists” which means grouped in braces and separated by commas.

    (c) Read off the coordinates of their two points of intersections. (Click on the graph, hold down the $\boxed{\text{Ctrl}}$ key and push the $\boxed{\text{d}}$ key, then click on the symbol that looks like a cross hairs $\boxed{-\!|\!-}$ in the new window and move the cursor over the points of intersection.)

    (d) `NSolve[f[x]==g[x]]`
    Notice that Mathematica can not solve this equation using the `NSolve` command, but you were able to read off approximate solutions using the graph in part c.

    (e) `Clear[f,g]`

Now use Mathematica and what you have learned so far in the following exercises.

A. Input the expressions $f = \frac{x-1}{x+1} - \frac{x+1}{x-1}$, $g = 2x^4 + 3x^3 - 7x^2 - 3x + 5$ and $h = \frac{4x+1}{(x-2)(2x+3)}$.

   (i) Simplify the expression named f. (Use the Simplify function.)

   (ii) Rewrite f as a single fraction. (Use the Together function.)

   (iii) Are the answers in the last two parts equivalent? if not, what is the difference?

   (iv) Factor g.

   (v) Rewrite h as sum or difference of two fractions. (Use the Apart function.)

   (vi) Clear the variables f, g and h.

B. Solve the following equations using both Solve and NSolve functions.

   (i) $15x^4 - 38x + 7 = 38x^3 - 22x^2$

   (ii) The system $2x = 7y - 4$ and $3x + 2y = 6$.

   (iii) The system $y = x^2 - 3x - 4$ and $y + x = 6 - x^2$.

C. Input the expressions $f(x) = \frac{x+1}{x-1}$ and $g(x) = x^2 - 2x + 3$ and perform the following steps.

(i) Draw the graph of $y = f(x)$ form $x = -1$ to $x = 3$.

(ii) Draw the graphs of $y = f(x)$ and $y = g(x)$ on the same coordinate system with the same domain.

(iii) Redraw these graphs by choosing an appropriate short domain so that the point of intersection of the two graphs is clearly visible.

(iv) Read off the coordinates of the point of intersection.

(v) Solve the equation $f(x) = g(x)$. Compare the real solution of this equation with the x-coordinate of the point of intersection.

(vi) Clear the two functions.

16. It is essential, both as a courtesy to future users, and to continued problem-free usage, to leave the computers as you find them. When you are done, close the software being used and/or log off properly.