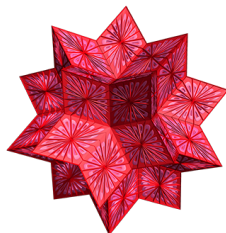Mathematics Computer Laboratory - Math 1200 - Version 14
Lab 11 - Review©



**Due**

You should only turn in exercises in this lab with its title and your name in `Title` and `Subtitle` font, respectively. Edit your document and use appropriate margins to get a polished and neat document.

1.    This lab is a collection of problems that can be done using previously studied Mathematica commands. You may need to review; if so, use earlier labs as a reference.

Perform each of the following exercises. It might be desirable to do some work by hand. In any case, be sure to explain your work by typing in or writing out necessary comments.

A.    Perform the indicated operations .

  (i) Multiply out $x(2x-1)^3(5-x)^2$ .

  (ii) Factor $-2x^3 + 7x^2 + 17x - 10$ .

  (iii) Combine $\frac{2x-1}{(x+1)(3-2x)} - \frac{x-1}{x(x+1)}$ into a single fraction and simplify, if possible.

  (iv) Rewrite $\frac{4-3x}{x(x-1)(2x-1)}$ into sum or difference of three fractions with denominators $x$ , $x-1$ and $2x-1$ .

B.    Solve the following system of equations using inverse matrices.

$$
\begin{array}{rrrrrrcr}
2u & -v & -w & -3x & +4y & -z & = & 18 \\
3u & +2v & +4w & -5x & -2y & +2z & = & -12 \\
u & +v & -2w & +5x & -y & -z & = & 18 \\
3u & +2v & +w & -6x & -2y & +2z & = & -7 \\
4u & -2v & -2w & +4x & -3y & +2z & = & -7 \\
u & -4v & -3w & -2x & +5y & -2z & = & 16 \\
\end{array}
$$

C.    Determine whether the exponential model $y = a\,e^{kx}$ or the power model $y = a\,x^b$ is the best fit to the following data. Then find the equation of that model.

| $x$ | 1.2 | 2.0 | 2.8 | 3.6 | 5.1 | 6.3 | 7.5 | 9.3 | 10.9 | 12.1 | 14.6 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $y$ | 5.31 | 4.92 | 4.51 | 4.19 | 3.59 | 3.18 | 2.69 | 2.37 | 2.02 | 1.79 | 1.40 |

D.   Find all solutions of the following equations or system of equations. Find the exact solutions whenever possible, otherwise find the approximate solutions.

(i) $-14x^3 + 57x^2 + 26x - 120 = 0$

(ii) $\begin{cases} y = \dfrac{x^2 - 2x - 3}{x^2 - 4} \\ 9x^2 + 16y^2 - 36x - 96y = 396 \end{cases}$

(iii) $3\cos 2x \, \sin x = 0.5x$

E.   Investigate the following limits. Decide if a limit has a value by using either numerical or graphical means. If a limit exists, find its value. If a limit does not exist, explain why.

(i) $\displaystyle\lim_{x \to 0} \frac{x - \sin x}{x^3}$

(ii) $\displaystyle\lim_{x \to 1} \frac{(x - 1)\cos x}{|x - 1|}$

(iii) $\displaystyle\lim_{x \to \infty} x^2 e^{-0.1x}$

F.   Consider the function $f(x) = 18x^4 - 8x^3 - 363x^2 + 840x + 20$.

(i) Find the intervals on which it is increasing and the intervals on which it is decreasing.

(ii) Find all its local extreme points.

(iii) Find the intervals on which its graph is concave up and the intervals on which it is concave down.

(iv) Find the inflection points of its graph.

(v) Graph this function. Try to show the results of the previous parts on the graph.

G.   Find an equation of a line tangent at two places to the graph of the function $y = x^4 + 5x^3 - 7x^2 - 29x + 31$. Graph this line and function on the same coordinate system.

H.   Evaluate $\displaystyle\int_0^{\pi} \cos^2 x \, dx$ using the limit of the Riemann sums.

I.   Evaluate the following integrals. If hand evaluation gives a nicer answer, include it.

(i) $\displaystyle\int_0^{\pi/2} x \sin 2x \, dx$

(ii) $\displaystyle\int \sqrt{\frac{x - 1}{x^5}} \, dx$   Check the answer, using Mathematica.

(iii) $\displaystyle\int 3x^2 (2024 - x^3)^{2025} \, dx$