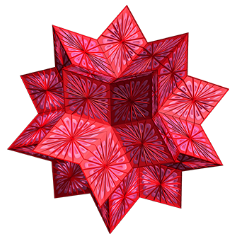
Mathematics Computer Laboratory - Math 1200 - Version 14

Lab 7 - Graphics©



Due

You should only turn in exercises in this lab with its title and your name in `Title` and `Subtitle` font, respectively. Edit your document and use appropriate margins to get a polished and neat document.

1. This lab reviews the `Plot` and `ListPlot` commands used previously and uses some new options for them. It discusses the secant lines of graphs and shows how to animate a series of graphs. The command `Table` is used for generating lists. Also, using the graphics package `Graphics`, the commands for drawing bar charts are introduced.

2. We have used the command `Plot` for a function defined by a formula in which we had to specify the independent variable and the domain as a list. Calling the function f and using x as its independent variable with the domain $[x\text{min}, x\text{max}]$, we can plot it using

   `Plot[f[x], {x,` $x$min, $x$max`}]`.

   The `Plot` command has many options. Some of these options are listed below. The variable x indicates the values on the horizontal axis and variable y indicates values of the vertical axis.

Option	**Purpose**	**Usage**
`PlotRange`	Specifies the displayed region.	`PlotRange->{{`$x$min, $x$max`}, {`$y$min, $y$max`}}`
`AxesOrigin`	Specifies the location of the crossing point of the axes.	`AxesOrigin->{`x-coord, y-coord`}`
`PlotStyle`	Specifies the style for each curve. It has several options.	`PlotStyle->{{`style option 1, style option 2, ...`}}`
`RGBColor`	An option in `PlotStyle` which specifies the color of the curve by mixing `Red`, `Green` and `Blue` colors.	`PlotStyle->{RGBColor[r, g, b]}` where r, g and b are numbers between 0 and 1 specifying the strength of the each of the colors red, green and blue, respectively, in the curve color.
`Dashing`	An option in `PlotStyle` which draws the curve as a sequence of dashed segments of specified length.	`PlotStyle->{Dashing[{`segment length`}]}`

   Now try the following.

   (a) `f[x_]:=Abs[x^3-8]` This defines $f(x) = |x^3 - 8|$.

   (b) `graph1=Plot[f[x], {x, -2, 3}]`

2. Continued.

(c) `Plot[f[x], {x, -2, 3}, PlotRange->{{-4, 4}, {-1, 15}}]`
The horizontal display interval does not have to be the same as the domain. Here the graph is displayed from $x = -4$ to $x = 4$ but the function is only plotted for $x = -2$ to $x = 3$.

(d) `Plot[f[x], {x, -2, 3},PlotRange->{{-4, 4}, {-1, 15}}, AxesOrigin->{1, 5}]`

(e)
```
Plot[f[x], {x, -2, 3},PlotRange->{{-4, 4}, {-1, 15}}, AxesOrigin->{1, 5},
                        PlotStyle->{RGBColor[1, 0, 0]}]
```

(f)
```
Plot[f[x], {x, -2, 3},PlotRange->{{-4, 4}, {-1, 15}}, AxesOrigin->{1, 5},
                        PlotStyle->{RGBColor[1, 0, 0], Dashing[{0.02}]}]
```

3. If a function or a relation is defined by a collection of points, we can graph it using the `ListPlot` command. This command accepts all options for the `Plot` command except, of course, the independent variable and its domain. In addition, you can specify the size of the points as an option in `PlotStyle`.

Option	Purpose	Usage
`PointSize`	An option for `PlotStyle` in `ListPlot` which sets the diameter of each point as a specified fraction of the whole graph.	`PlotStyle->{PointSize[`diameter`]}` A good value for the point size depends on the size of the displayed region but a safe value is any number between 0.01 and 0.03.

Try the following.

(a) `gfunc={{-2,-3}, {-1.5,10}, {-1,6}, {0,10},{1,8},{1,12}, {2,3}, {3, 4}}`

(b) `ListPlot[gfunc]`

(c) `graph2=ListPlot[gfunc, PlotStyle->{PointSize[0.02]}]`

4. Two or more graphs can be combined using the `Show` command. The command `PlotStyle` and some other `Plot` options can also be used with the command `Show`.

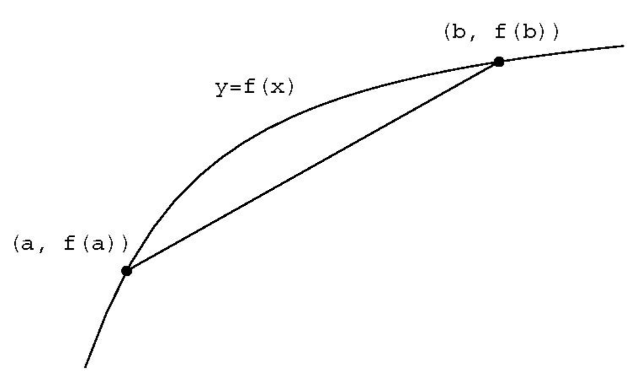
(a) `Show[{graph1, graph2}]`
Notice that the point $(-2, -3)$ is no longer displayed when the two graphs are combined. The solution to this problem is to specify the region to be displayed.

(b) `Show[{graph1, graph2}, PlotRange->{{-3, 4}, {-4, 15}}]`

(c) `Clear[f, gfunc, graph1, graph2]`

5. Consider the function $y = f(x)$. A line through two points on the graph of $y = f(x)$ is called a secant line. The slope of the secant line through points $(a, f(a))$ and $(b, f(b))$ is $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(b) - f(a)}{b - a}$ and the equation of the secant line after using the point $(a, f(a))$ as (x_1, y_1) in the point-slope formula $y - y_1 = m(x - x_1)$ simplifies to

$$y = f(a) + \frac{f(b) - f(a)}{b - a}(x - a).$$



Try the following.

5. Continued.

(a) `f[x_]:=1+2Cos[x]`

(b) `m=(f[Pi/2]-f[-Pi/4])/(Pi/2-(-Pi/4))` The slope of the secant line through points $(-\pi/4, f(-\pi/4))$ and $(\pi/2, f(\pi/2))$.

(c) `sl[x_]:=f[Pi/2]+m(x-Pi/2)` The equation of the secant line through points $(-\pi/4, f(-\pi/4))$ and $(\pi/2, f(\pi/2))$.

(d)
```
Plot[{f[x], sl[x]}, {x, -Pi/2, Pi},
                        PlotStyle->{{RGBColor[1, 0, 0]}, {RGBColor[1, 0, 1]}}]
```

(e) Read off the coordinates of the points of intersections of the red and purple curves and verify that they are points $(-\pi/4, f(-\pi/4))$ and $(\pi/2, f(\pi/2))$. (Right click on the graph and choose Drawing Tools (or Get Coordinates) in the window that opens up. In the new window, click on the symbol that looks like a cross hairs [–|–] and move the cursor over the graph.)

(f) Clear[f, m, sl]

6. We can use Mathematica to generate and display a series of graphs: for example, a function with a series of secant lines through a fixed point on its graph.

(a) `f[x_]:=(x-0.25)^3+1.125`

(b) `m[t_]:=(f[t]-f[1])/(t-1)` This gives the slope of any secant line through the fixed point $(1, f(1))$ and the arbitrary point $(t, f(t))$.

(c) `sl[x_]:=f[1]+m[t](x-1)` This gives the equation of any secant line through the fixed point $(1, f(1))$ and the arbitrary point $(t, f(t))$.

(d) We now want to graph $y = f(x)$ and $y = sl(x)$ in red and green for various values of t. Suppose we want to obtain the graphs

```
Plot[{f[x], sl[x]}, {x, -0.25, 2.5}, PlotRange->{{0, 2.25}, {-0.5, 7}},
    PlotStyle->{{RGBColor[1, 0, 0]}, {RGBColor[0, 1, 0]}}]
```
(Do not type this in yet!)

starting with $t = 2$ and reducing t by 0.1 and stopping at $t = 1.1$.

We can generate a series of graphs for t values in a given range using the `Table` command. The usage of this command is as follows

`Table[`expression or graph`,` {parameter, start value, stop value, step size} `]`.

Input and execute the following.

```
Table[Plot[{f[x], sl[x]}, {x, -0.25, 2.5}, PlotRange->{{0, 2.25}, {-0.5, 7}},
                PlotStyle->{{RGBColor[1, 0, 0]}, {RGBColor[0, 1, 0]}}],
        {t, 2, 1.1, -0.1}]
```

(e) We can animate these graphs. All is needed is to add `ListAnimate` command at the beginning and brackets to encapsulate the table. Go back to the last cell and add `ListAnimate[` at the beginning and `]` at the end, as shown below, and execute it.

(e) Continued.

```
ListAnimate[Table[Plot[{f[x], sl[x]}, {x, -0.25, 2.5}, PlotRange->{{0, 2.25},
               {-0.5, 7}},PlotStyle->{{RGBColor[1, 0, 0]}, {RGBColor[0, 1, 0]}}],
               {t, 2, 1.1, -0.1}]]
```

Notice that you get a window that shows an animation or a "movie" of the graphs. You can control the animation, like its speed and direction, through buttons which are at the top of that window.

7. Let's examine the slopes of the secant lines in part 6 as the value of t approaches 1. Notice that we cannot just find `m[1]` since it is undefined: $m(1) = \frac{f(1)-f(1)}{1-1} = \frac{0}{0}$. However, we can find the value it approaches by forming tables.

(a) `Table[{t, m[t]}, {t, 2, 1.1, -0.1}]//TableForm` In each ordered pair or list, the first number is the t-value and the second is its corresponding m-value. At $t = 1.1$, the slope is $m = 1.9225$.

(b) To get values of m for t values closer to 1, form another table from $t = 1.1$ to $t = 1.01$ decreasing in steps of 0.01.
`Table[{t, m[t]}, {t, 1.1, 1.01, -0.01}]//TableForm` At $t = 1.01$, the slope is $m = 1.7101$.

(c) To get values of m at t values even closer to 1, form another table from $t = 1.01$ to $t = 1.001$ decreasing in steps of 0.001.
`Table[{t, m[t]}, {t, 1.01, 1.001, -0.001}]//TableForm` At $t = 1.001$, the slope is $m = 1.68975$.

The value $m = 1.68$ seems to be a good approximation for the value which m approaches as t gets close to 1.

(d) The equation of the line through the point $(1, f(1))$ with slope $m = 1.68$ is $y = f(1) + 1.68(x - 1)$. Evaluate the following.

```
tl[x_]:=f[1]+1.68(x-1)
```

(e) Let's redo the last animation and also include this new line.

```
ListAnimate[Table[Plot[{f[x], sl[x], tl[x]}, {x, -0.25, 2.5},
                  PlotRange->{{0, 2.25}, {-0.5, 7}},
                  PlotStyle->{{RGBColor[1, 0, 0]}, {RGBColor[0, 1, 0]},
                              {RGBColor[0, 0, 1]}}],
               {t, 2, 1.1, -0.1}]]
```

(d) `Clear[f, m, sl, tl]`

8. Charts and in particular bar charts are frequently used to convey information. Bar charts are also used to develop the third main idea in calculus: Riemann integration. If you are using Mathematica version 6 or earlier, first do part c. Input and evaluate the following.

(a) `list1={5, 2, -3, 4, 6.5, -3}`

(b) `BarChart[list1]` You get 6 bars over $x = 1$, $x = 2$, $\cdots$, $x = 6$ with the corresponding heights of 5, 2 and so on, as given in `list1`.

(c) The `GeneralizedBarChart` command becomes available after the `BarCharts` package is loaded. Note: In Mathematica versions 7-11, you will get a message stating that the command is obsolete. But, there is no exact replacement for it and it will still work!

`` Needs["BarCharts`"] `` There are no blank spaces. The symbol ` is on the same key with ~ and in most keyboards that key is on the far left side next to the key for number 1.

8. Continued

(d) In addition to the height of a bar, we can specify where it should be displayed and what its width should be. The command `GeneralizedBarChart` is used for this purpose and its usage is

`GeneralizedBarChart[{{position1, height1, width1},{position2, height2, width2}, ⋯}]`

Try the following.
`GeneralizedBarChart[{{-1, 3, 1/2}, {2, -1, 2}, {4, 5, 1}}]` You should get three bars centered at $x = -1$, $x = 2$ and $x = 4$ with corresponding height 3, -1 and 5 and the width of the bars are $1/2$, 2 and 1 respectively. Note: Be sure to load the package `BarCharts` before using the command `GeneralizedBarChart`.

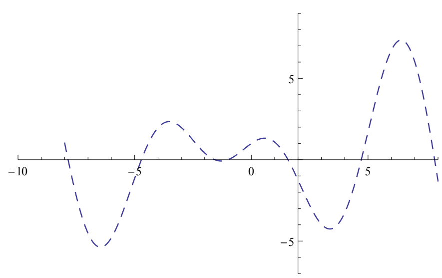
(e) In many instances it is convenient to generate the ordered triples {position, height, width} using a table command.

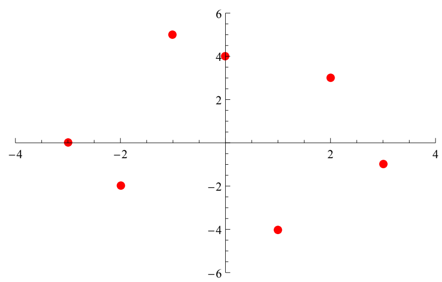
```
list2=Table[{2n, 2n/(2n-1), (9-n)/10}, {n, 1, 5, 1}]
GeneralizedBarChart[list2]
```

(f) `Clear[list1, list2]`

Perform each of the following exercises. Explain your work by typing in necessary comments in text cells.

A. Produce graphs which best resemble each of the following. The one in the left is the graph of the function $f(x) = (x+1)\cos x$. The color of the points in the graph in the right is **red**.





B. Consider the function $f(x) = x^2$. Find the equation of the secant line through the fixed point $(1, f(1))$ and arbitrary point $(t, f(t))$. For the values from $t = -1.5$ to $t = 0.9$ in steps of 0.1, use the `Table` command to plot this function and its secant lines in two different colors on the same coordinate system using `PlotRange->{{-1.75, 2}, {-1, 3.25}}`. Make a movie of the graphs using the `ListAnimate` command. (If printing a document, only one graph in the animation window will print.)

C. Again, consider the function $f(x) = x^2$ and its secant lines through the point $(1, f(1))$. Use the `Table` command to approximate the value the slopes of these secant lines approach as t approaches 1 from the **left** and the **right** of it. Your t values should **at least** get within $\pm$**0.01** of $t = 1$. Find the equation of the line through the point $(1, f(1))$ with the slope the approximate value you just found. Plot $f(x) = x^2$ and this line on the same coordinate system. (No animation of secant lines!)

D. Generate bar charts with the following properties.

(i) 5 bars positioned at $x = 1$ through $x = 5$ with heights 5 through 1, respectively . Use the `BarChart` command.

(ii) 4 bars positioned at $x = -3, -1, 2, 4$ with corresponding heights $-2, -1, 1$ & 2 and widths of $1/2$, 1, 0.3 & 1.5 respectively.

(iii) Bars positioned at $\frac{n^2}{n+1}$ with corresponding heights $(n-2.5)^3$ and widths $\frac{n}{2n-1}$, for each number n from 1 to 6 in steps of 1. Use the `Table` command.