

College Algebra

Fun Problems

1. Good coins weigh 10 grams, bad ones 9 grams . Given 4 coins and a scale (not a balance, but a true scale), determine which are which in only 3 weighings.
2. Find the *exact* value of $\sqrt{2 + \sqrt{2 + \sqrt{2 + \cdots}}}$.
3. Prove that $\sqrt[3]{2 + \sqrt{5}} + \sqrt[3]{2 - \sqrt{5}}$ is an integer.
4. A function f satisfying $f(x + y) = f(x) + f(y)$ for all real numbers x and y is said to be an additive function. Prove that if f additive, then there is a number m such that $f(t) = mt$ for all rational numbers t .
5. We are given a set of 201 different numbers with the property that the sum of any 100 of them is less than the sum of the remaining 101. Prove that all of the numbers are positive.
6. Sam Slugger had a better batting average than Hank Homer in the first half of the baseball season. For the second half of the baseball season, Sam's batting average was also higher than Hank's batting average. Does it follow that the overall batting average of Sam is better than the overall batting average of Hank? Why or why not?
7. Find the four square roots of the matrix $\begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix}$.
8. Suppose the size of one water lily doubles every 3 days and it will cover a certain pond in 30 days. How long will it take two of these lilies to cover the pond?
9. Find the sum of all *digits* in the integers from 1 to 999,999.
10. Sally walked 4 miles north, then 2 miles east, 1 mile south, $\frac{1}{2}$ mile west, $\frac{1}{4}$ mile north, and so on. If she continued this pattern indefinitely, how far from her initial point did she end?