Image reconstruction algorithm for a rotating slat collimator

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Citation: Medical Physics 29, 1406 (2002); doi: 10.1118/1.1485057
View online: http://dx.doi.org/10.1118/1.1485057
View Table of Contents: http://scitation.aip.org/content/aapm/journal/medphys/29/7?ver=pdfcov
Published by the American Association of Physicists in Medicine

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I. INTRODUCTION

In SPECT (single photon emission computed tomography) a parallel-hole collimator is routinely used in conjunction with a large rectangular or circular gamma detector. Currently most investigators of CdZnTe detector build a small area square detector due to the high cost of CdZnTe. This design significantly limits the field-of-view of the detector. With the same detection area we propose using a narrow-and-long detector which can image a much larger field of view. The price we pay to get a larger field-of-view is employing an additional spinning motion.

In the development of a novel, CdZnTe strip detector, a parallel-hole collimator is not efficient in collecting data for a strip detector. It is suggested that a set of parallel slats be used to collimate the photons. Figure 1 illustrates the detector and collimator geometry.

Compared with a regular rectangular detector, the strip detector is rather narrow, and it can be treated as a linear detector. In order to acquire a complete data set the detector has to spin by itself and rotate around the object (patient), as illustrated in Fig. 2. We refer to each detector center position as a SPECT position or a SPECT view. The minimum requirement for obtaining a complete data set is 180° of rotation of the detector assembly around the patient and the detector spinning 180° at each SPECT view.

A rotating slit collimation has two major drawbacks—the additional spinning motion and increased scattered photon. Scatter issue is a major concern in our detector development. One advantage of using CdZnTe instead of using NaI(Tl) crystals is that a NaI(Tl) detector has about 10% energy resolution at 140 keV while we can reach 3.65% energy resolution. Thus, most of the scattered photons can be rejected by using a CdZnTe detector and the projection data can be approximated as scatter-free.1,2

As illustrated in Fig. 1(a), each detector cell measures a weighted planar integral of the isotope distribution within a slice of the object in the field of view of the detector cell. This slice is the region between two adjacent collimation slats, as shown in Fig. 1(b). The weighting in the planar integral is inversely proportional to the distance, r, from the location of interest to the detection cell, see Fig. 1(a). This 1/r sensitivity factor is also observed in a fan-beam imaging geometry, in which r is the distance from the object to the fan-beam focal point.3 In addition to this 1/r weighting factor, the detection solid angle also affects the sensitivity, which can be approximately characterized by a cos α factor for a strip detector. The angle α is defined in Fig. 1(a). The weighting factor (cos α)/r is still approximate as it assumes a perfect collimation and neglects septal penetration and some other practical considerations. A perfect collimation implies that the collimator blurring function is a delta function. With such ideal collimation, a detector can only detect photons emitted from the two-dimensional (2D) plane that contains the detection cell and is parallel to the slats, as if the slats had infinite size and can even cut through the object, as shown in Fig. 3. A more general expression of the sensitivity function has been derived in Lodge et al.4

Without this (cos α)/r weighting factor, the resultant planar integrals are in the form of the well-known Radon transform in the three-dimensional (3D) space. A Radon inversion formula can be used to reconstruct the 3D image from the unweighted planar integrals.5

Investigation of a rotating slit collimator has been pursued by Webb et al.6,7 Lodge et al.4 and Britten and Klie.8 Each used a large circular gamma detector combined with a rotating slit collimator for planar imaging. A conventional 2D filtered backprojection algorithm was used to form the 2D planar images. Later, Lodge et al. used the rotating slit
collimator to obtain 3D SPECT images. They made an attempt to reduce the $1/r$ weighting factor effect by using the geometric mean of two opposing projection measurements.

The purpose of this paper is to develop a method that can be used to reconstruct 3D images from $(\cos \alpha)/r$ weighted planar integrals for our strip detector application. The inspiration for our proposed method is a cone-beam reconstruction algorithm, which converts the derivative of cone-beam data into a derivative of parallel planar integrals (Radon data).

The goal of this paper is not to evaluate the performance of the CdZnTe SPECT system that is under development. The goal of this paper is to develop a reconstruction algorithm for an ideal rotating slat SPECT imaging system. By an ideal imaging system, we mean:

\begin{itemize}
  \item A perfect collimation uses impractical large slats that cut through the object, ensuring that the measurements are parallel planar integrals. With this ideal collimation, the collimator blurring function is a delta function and is not a function of the source distance.
  \item There is no photon attenuation within the patient. Therefore, no scatter, attenuation, and collimator blurring effects are considered in our computer simulations.
\end{itemize}

II. METHODS

The strategy behind our method is to remove the $1/r$ weighting factor in the measurements from a slat collimator through hardware modification and application of an image reconstruction algorithm.

A. Hardware modification

We suggest tilting all of the parallel slats for a small angle, $\delta$, toward one end of the detector assembly as shown in Fig. 4. The selection of this small angle will depend on the spatial resolution of the associated detector. Another modifi-
cation is an increase in spin angle to 360°, instead of 180°. After the slats are tilted by δ, the projection data values are different when the detector spin angles are 180° apart.

If we spin the detector 360°, data are acquired twice at each detector cell location, specified by t, θ, and φ. If δ=0 these two measurements are identical. If δ>0, these two measurements are different. We denote the first measurement with the slat tilted toward the positive t direction as \( g_{\theta,\phi}^{up}(t,\delta) \) and the second measurement with the slat tilted toward the negative t direction as \( g_{\theta,\phi}^{down}(t,\delta) \).

**B. Reconstruction algorithm**

Before developing a new reconstruction algorithm, let us first review the Radon inversion formula. Let the unweighted planar integral be

\[
R_{\theta,\phi}(t) = \int\int f(r,\alpha,t) r \, dr \, d\alpha.
\]

and \( R_{\theta,\phi}(t) \) is the Radon transform of \( f \) in a three-dimensional space and \( f \) is defined in a local coordinate system shown in Fig. 5. The partial derivative of \( R_{\theta,\phi}(t) \) with respect to \( t \) is given as

\[
\frac{\partial}{\partial t} R_{\theta,\phi}(t) = \int\int f'_r(r,\alpha,t) r \, dr \, d\alpha.
\]

A well-known analytical algorithm based on the Radon inversion formula is used to reconstruct a 3D image from unweighted planar integrals.

\[
f_{\text{global}}(x,y,z) = -\frac{1}{4\pi^2} \int_0^\pi \int_0^\pi R_{\theta,\phi}^\prime \times (t l_{-x} \sin \theta \cos \phi + y \sin \theta \sin \phi + z \cos \theta) \sin \theta d\theta d\phi,
\]

where \( f_{\text{global}}(x,y,z) \) is the object expressed in a global Cartesian coordinate system, \( R_{\theta,\phi}(t) \) is the unweighted planar integral, \( t \) is the linear coordinate along the detector assembly, \( \phi \) is the rotation angle, \( \theta \) is the spinning angle, and the z axis is the axis of rotation, which corresponds to \( \theta=0 \).

Now consider the imaging system developed in Sec. II A. Let the 3D object to be imaged be expressed in a local cylindrical coordinate system as \( f(r,\alpha,t) \), where the \( t \) axis is along the axis of the strip detector and \( \alpha=0 \) indicates the line perpendicular to the slat’s longer side. Here, index \( t \) indicates the detection cell location along the strip detector and \( t=0 \) corresponds to the location on the central cell of the strip detector. If the parallel slats are tilted by \( \delta \) in the positive \( t \) direction the first measured weighted planar integral is given by

\[
g_{\theta,\phi}^{up}(t,\delta) = \int\int \frac{\cos \alpha}{r} f(r,\alpha,t+r \cos \alpha \tan \delta) r \, dr \, d\alpha.
\]

(4)

After the strip detector spins 180° the second measurement is

\[
g_{\theta,\phi}^{down}(t,\delta) = \int\int \frac{\cos \alpha}{r} f(r,\alpha,t-r \cos \alpha \tan \delta) r \, dr \, d\alpha.
\]

(5)

Even if the detector spins 360°, we still restrict the range of \( \theta \) to \( 0 \leq \theta < \pi \). In this way there is no ambiguity between Eqs. (4) and (5).

We form a new set of modified projections

\[
g_{\theta,\phi}(t,\delta) = \left[ g_{\theta,\phi}^{up}(t,\delta) - g_{\theta,\phi}^{down}(t,\delta) \right] \frac{1}{2 \tan \delta}
\]

(6)

for \( 0 \leq \theta < \pi \). This modified projection data set will be used in the image reconstruction algorithm.

Evaluating the right-hand side of Eq. (6) yields
\[ g_{\theta, \phi}(t, \delta) = \int \int \frac{r \cos^2 \alpha f(r, \alpha, t + r \cos \alpha \tan \delta) - f(r, \alpha, t - r \cos \alpha \tan \delta)}{2r \cos \alpha \tan \delta} \, dr \, d\alpha, \] 

that is,

\[ g_{\theta, \phi}(t, \delta) = \int \int \cos^2 \alpha f'(r, \alpha, t) \, dr \, d\alpha = \frac{\partial}{\partial t} \int \int \cos^2 \alpha f(r, \alpha, t) \, dr \, d\alpha. \] 

It is observed that the modified projection \( g_{\theta, \phi}(t, \delta) \) does not contain the \( 1/r \) weighting factor that exists in the original measurements \( g_{\theta, \phi}^{\text{up}}(t, \delta) \) and \( g_{\theta, \phi}^{\text{down}}(t, \delta) \).

However, by comparing Eq. (8) with Eq. (2), we cannot directly use the Radon inversion formula (3) to reconstruct the image due to an extra factor \( \cos^2 \alpha \) in Eq. (8). Modifying Eq. (3), we suggest the following approximate method to reconstruct the image:

\[ f_{\text{global}}(x, y, z) = -\frac{1}{4\pi^2} \int_0^\pi \int_0^\pi g_{\theta, \phi}' \times (t|_{t=x \sin \theta \cos \phi + y \sin \theta \sin \phi + z \cos \theta}) \times \frac{1}{\cos^3 \alpha} \sin \theta \, d\theta \, d\phi, \] 

where the backprojection weighting factor \( 1/\cos^2 \alpha \) attempts to compensate for the \( \cos^2 \alpha \) in Eq. (8). The angle \( \alpha \) is the angle of the reconstruction location \((x, y, z)\) deviated from the line orthogonal to the detector surface at each backprojection direction.

### III. POINT RESPONSE EVALUATION

In this section, we evaluate the point response of the projection/backprojection pairs. The first pair consists of the unweighted planar projection (i.e., the Radon transform) and unweighted planar backprojection. The second pair involves \( \cos^2 \alpha \) weighted projection and \( 1/\cos^2 \alpha \) weighted backprojection. It is well-known that the point response function for the unweighted planar projection/backprojection is shift-invariant, and a second-order derivative operator can be used to exactly reconstruct the image. It is hoped that the spatially varying point response function for the weighted projection/backprojection can be accurately approximated by the point response function of the unweighted planar projection/backprojection. Therefore, the image can be accurately reconstructed by back projecting the second-order derivative function.

We first put the test point source at the center of the field of view, that is \((x_1, y_1, z_1) = (0,0,0)\). In the second study, the test point source was located 125 mm away from the field-of-view center, at a corner position \((x_2, y_2, z_2) = (72,72,72)\) mm.

In this point response evaluation study, the projection data were generated by directly assigning the projection data as \((\cos \alpha)/r\), where \(\alpha\) and \(r\) were determined by the point source and detector positions. The point source was an ideal delta function (i.e., the size of the point source is zero), while the detector was discretized with a finite detection cell size. Ideal collimation was assumed, and the projection value was zero if that particular detection cell did not directly see the point source. Thus, the dimensions of the slat collimator are irrelevant.

Figure 6 shows the three orthogonal profiles of the point response function for the center test point source, and Fig. 7 shows the three orthogonal profiles of the point response function for the corner test point source. It is observed that when the point source is at the center, the point response functions for unweighted and weighted pairs are almost identical. When the point source is at the corner, some discrepancies can be visualized in the \(x\) and \(y\) directions. The profiles are displayed in log scale in order to visualize the small discrepancies which are difficult to detect in linearly scaled plots. The blurring in the point response functions (see Figs. 6 and 7) was not caused by the finite size of the point source or the collimator blurring. The blurring seen in Figs. 6 and 7 is called the tomographic point response function, which is generated by first giving a delta point source in the image space, then performing the projection procedure, and finally performing the backprojection procedure. It is supposed to

![Fig. 6. Three orthogonal profiles of the weighted (—) and unweighted (—) projection/backprojection point response functions when the test point source is located at the center. The vertical axes are in log scale.](image-url)
get a blurred image of the point source. It is well known that if the projection is a pure planar integral without any weighting factors, the backprojection of a second-derivative filtered data will give the original image back. The purpose of Figs. 6 and 7 is that if the weighted tomographic point response function is very close to the unweighted tomographic point response function, the second-derivative filtering is expected to give a very close approximation of the original image.

IV. COMPUTER SIMULATION

The purpose of the computer simulation is to observe whether the reconstruction algorithm works when data are generated with a realistic (nonideal) slat collimator. Even though the algorithm was derived by assuming ideal collimator, the Monte Carlo simulator generated data with a finite-sized slat and the collimator blurring function was source distance dependent. The Monte Carlo data were also noisy. A Monte Carlo simulation code GEANT was used to generate projection data. GEANT is a detector description and simulation tool initially developed by CERN, which is a European organization that performs nuclear research.

A point source was used in this computer simulation study. The point source was a solid ball with a radius of 1 cell size (i.e., 1.8 mm). There were 64 SPECT-views over 360° and at each SPECT-view there were 64 spin-views over 360°. The simulated projections were stored in a 128 × 64 × 64 array. The detector was one-dimensional and comprised 128 detection cells. The cell size in the axial direction was 1.8 mm. The collimator slat angle δ was 1°. The photon energy was assumed to be 140 keV. No scattered photons were accepted. The phantoms were placed a few voxels off the detector rotating/spinning center. The distance between the detector and the rotation center was 17 cm. The voxel size in the reconstructed image was 1.8 × 1.8 × 1.8 mm³. The dimensions of the CdZnTe detection cell and tungsten collimation slat are given in Fig. 8. However, the reconstruction algorithm still assumes a perfect collimation as illustrated in Fig. 3.

Figure 9(a) shows a set of projection data at the first SPECT-view for the point source simulation. The horizontal axis is the linear direction of the strip detector (128 cells). The vertical axis is the spin angle (64 positions over 360°). Figure 9(b) shows three orthogonal views of the reconstructed image of a hot point source. No pre- or postfiltering was performed on the images.

The reconstruction code was written in IDL, which is an Interactive Data Language developed by Research Systems, Inc., Boulder, Co. The basic steps in the code are as follows:

![Fig. 7. Three orthogonal profiles of the weighted (—) and unweighted (---) projection/backprojection point response functions when the test point source is located at the corner. The vertical axes are in log scale.](image1)

![Fig. 8. Detector and collimator slat dimensions used in computer simulation and experimental prototype detector. The detector cells are made of CZT crystals and the collimator slats are made of tungsten.](image2)

![Fig. 9. Point source computer simulation. (a) Projection data from the first SPECT-view. The projection data are generated via a Monte Carlo code. The horizontal axis is the detection cell coordinate on the strip detector and the vertical axis is the spin angle. This display is called a sinogram. (b) Three orthogonal views cut through the point source in the reconstructed image volume. The reconstruction procedure is outlined at the end of Sec. IV.](image3)
For every SPECT-view $\phi$ do
  For every spin angle $\theta$: $0 \leq \theta < \pi$ do
    For every cell position $t$, calculate $g_{\theta,\phi}(t, \delta)$ defined in equation (6)
    For every cell position $t$, take derivative $g'_{\theta,\phi}(t, \delta)$ with respect to variable $t$
    Scale by $\sin \theta$, obtaining $g_{\theta,\phi}(t, \delta)\sin \theta$
    3D backprojection with weighting $1/\cos^2 \alpha$
  end do
end do

V. PHANTOM EXPERIMENT

A prototype of a solid-state detector was built utilizing approximately the same parameters established for the computer simulation. A Hoffman brain phantom was used for data acquisition, with 925 MBq (25 mCi) of Tc-99m. The total data acquisition time was 2 h.

There were 180 SPECT-views over 360° and at each SPECT-view there were 256 spin-views over 360°. The acquired projections were stored in a 192$\times$256$\times$180 array. The detector was one-dimensional and comprised 192 detection cells. The cell size in the axial direction was 1.8 mm. The collimator slat angle $\delta$ was 1°. The energy window was centered at 140 keV with a 12% window width. The distance between the detector and the rotation center was 20 cm. The voxel size in the reconstructed image was $1.8\times1.8\times1.8$ mm$^3$.

Figure 10 shows four slices of the reconstructed image of the Hoffman phantom. The reconstruction code was the same as the one used in the computer simulation. This phantom study involved scatter, attenuation, and collimator blurring effects, which were not considered in the reconstruction algorithm development.

VI. DISCUSSION

In this paper projections measured with a slat collimator are described as $(\cos \alpha)/r$ weighted planar integrals, and $r$ is the distance from the source to the detection cell and $\alpha$ is the angle of the point of interest deviated from the line orthogonal to the detector surface. In practice, this weighting factor is only an approximation. The assumption that the measurement is confined in a plane (see Fig. 3) is impractical. It is known that the detector resolution worsens as the object is placed farther away from the detector. In a regular SPECT camera with a parallel-hole collimator, the full width at half maximum value of the collimator point response function is a linear function of the distance from the point source to the detector. A similar blurring behavior of the distance dependent spatial resolution degradation is expected for the strip detector and the measurement sensitivity function can be approximated as $(\cos^2 \alpha)/r$. This collimator blurring effect is not considered in this paper. Only small modification is needed in the proposed algorithm if it is desired to implement a reconstruction algorithm for the sensitivity function $(\cos^2 \alpha)/r$. Photon attenuation and scattering effects within the patient’s body are also not included in this paper.

One advantage of using the parallel slats as a collimator is that the measurement of weighted planar integrals has many more photon counts than the measurement of line integrals obtained using a parallel-hole collimator. It must be pointed out that the increased sensitivity does not automatically imply an improved image due to degradation during the reconstruction procedures, especially for imaging large objects.

There are some disadvantages of the proposed methods. Even though the slats are parallel, we require a 360° spin of the collimator rotation instead of 180°. Since there is a spatially varying $1/\cos^2 \alpha$ factor in the backprojector, Marr’s efficient backprojector is not applicable.

With the suggested filtered backprojection reconstruction algorithm, the 3D image is reconstructed by taking finite differences of the projection data and by applying back-projection. The filtering step is performed by the difference operation. In a regular 2D filtered backprojection algorithm, ramp filtering is used. Comparing the difference operation with the ramp-filtering operation, the difference operates locally while the ramp filtering operates globally in the sense that a local operator has a very short convolution kernel, while a global operator has a very long convolution kernel. With a long convolution kernel, measurement errors propagate to the reconstruction point from locations far away. With a short convolution kernel, measurement errors do not propagate farther than half of the kernel length. Therefore, with a parallel-slat collimator one can use a small detector to image a small region of interest (ROI) without incurring any truncation artifacts. The proposed collimator and reconstruction algorithm may be applicable when imaging small ROIs in a large object, which is usually referred to as local tomography. The strip detector only needs to be long enough to cover the ROI. It does not need to cover the entire object. As a result, it is immune from truncation errors. This
local tomography method will be evaluated in our future work.

The small tilt angle $\delta$ is dependent on the system spatial resolution and may need to be determined experimentally. As a rule of thumb, a higher system spatial resolution requires a smaller tilt angle $\delta$.

ACKNOWLEDGMENT

The authors wish to thank Sean Webb of the University of Utah for English editing.

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