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A three-dimensional ray-driven attenuation, scatter and geometric response correction technique for SPECT in inhomogeneous media

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Abstract. The qualitative and quantitative accuracy of SPECT images is degraded by physical factors of attenuation, Compton scatter and spatially varying collimator geometric response. This paper presents a 3D ray-tracing technique for modelling attenuation, scatter and geometric response for SPECT imaging in an inhomogeneous attenuating medium. The model is incorporated into a three-dimensional projector–backprojector and used with the maximum-likelihood expectation-maximization algorithm for reconstruction of parallel-beam data. A transmission map is used to define the inhomogeneous attenuating and scattering object being imaged. The attenuation map defines the probability of photon attenuation between the source and the scattering site, the scattering angle at the scattering site and the probability of attenuation of the scattered photon between the scattering site and the detector. The probability of a photon being scattered through a given angle and being detected in the emission energy window is approximated using a Gaussian function. The parameters of this Gaussian function are determined using physical measurements of parallel-beam scatter line spread functions from a non-uniformly attenuating phantom. The 3D ray-tracing scatter projector–backprojector produces the scatter and primary components. Then, a 3D ray-tracing projector–backprojector is used to model the geometric response of the collimator.

From Monte Carlo and physical phantom experiments, it is shown that the best results are obtained by simultaneously correcting attenuation, scatter and geometric response, compared with results obtained with only one or two of the three corrections. It is also shown that a 3D scatter model is more accurate than a 2D model.

A transmission map is useful for obtaining measurements of attenuation and scatter in SPECT data, which can be used together with a model of the geometric response of the collimator to obtain corrected images with quantitative and diagnostically accurate information.

1. Introduction

Accuracy of SPECT images is degraded by physical effects of attenuation, Compton scatter, detector response and patient movement. Compensation for attenuation (Tsui et al 1989) has been studied at great length, and several methods have been proposed. However, correcting for attenuation only is not sufficient because the erroneous information given by scattered photons tends to produce an over-compensation. This is seen as the major source of apparent over-correction in the inferior wall of the left ventricle of the heart, which is associated with hepatic uptake (King et al 1996). Consequently, it is necessary to correct simultaneously for attenuation

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and scatter. It has been shown that when accurate scatter models are used the over-correction in the heart is improved (Tsui et al 1994) and the signal to noise ratio (SNR) of small cold lesions is also improved (Beekman et al 1996, 1997a, Hutton et al 1996, Hutton 1997). In addition, it has been known for some time that the geometric response of the collimator produces a spatially varying partial volume effect in SPECT images. It was found, for example in dynamic cardiac SPECT (Gullberg et al 1998), that the correction for geometric response improves the partial volume effect and reduces the bias in estimated kinetic parameters (Kadrmas et al 1999). While some methods perform presubtraction of the scatter component from projection data before the reconstruction process, linear algebraic methods allow the incorporation of attenuation, scatter and geometric response corrections in the reconstruction process. This is done by utilizing the projector–backprojector (transition matrix) which models the acquisition process in order to produce improved qualitative and quantitative SPECT images.

A class of widely used and widely studied scatter compensation methods is based on the estimation of the scattered component in the photopeak projection data and subsequent subtraction or deconvolution of the scatter contribution from the measured projection data. Many of the subtraction-based scatter compensation methods use multiple energy window acquisition methods (DeVito et al 1989, Frey et al 1992, Gagnon et al 1989, Halama et al 1988, Hamill and DeVito 1989, Jaszczyk et al 1984, 1985, King et al 1992, Koral et al 1988, 1990, Lowry and Cooper 1987, Ogawa et al 1991) to estimate the scatter contribution. The estimated scatter component can be subtracted from the photopeak projection data to obtain the primary projection data. Then SPECT reconstruction methods without scatter compensation can be used. Scatter compensation methods in this class are fast and simple but increase the noise in the reconstructed images. The deconvolution methods use kernels that are obtained from physical measurements using a gamma-camera (Axelsson et al 1984, Floyd et al 1985b, Meikle et al 1994, Msaki et al 1987, 1989). Usually, the scatter component is deconvolved from the projection data before reconstruction. Deconvolution or restoration filtering methods can also be used to correct for geometric response. The restoration filtering is implemented in a filtered backprojection algorithm (Gilland et al 1988, King et al 1983, 1984, Ogawa et al 1988) where the blurring is approximated by a spatially invariant blurring function. Another approach uses the frequency–distance principle (Elmbt and Walrand 1993, Glick et al 1994, Lewitt et al 1989, Xia et al 1995) which incorporates the distance-dependent collimator blurring into the Fourier transform of the sinogram.

Several techniques have been developed for calculating the transition matrix (for scatter and geometric response modelling) so that it is a function of the collimator geometry and the anatomy being imaged:


(c) Another technique is referred to as slab-derived scatter estimation. This technique first calculates and stores the scatter response tables for a point source behind slabs of a range of thicknesses, and then tunes the model to various object shapes (Beekman et al 1993, 1996, 1997b, Beekman and Viergever 1995, Frey and Tsui 1993, Frey et al 1993). A table that occupies only a few megabytes of memory is sufficient to represent this scatter model for fully 3D reconstruction.

(d) Another method is based upon the integration of the Klein–Nishina formula in non-uniform media (Cao et al 1994, Riauka and Gortel 1994, Riauka et al 1996, Wells et al 1998). Like Monte Carlo techniques, this technique requires large data storage capacities and long computation times.

(e) Another approach, which improves computation and storage requirement, uses an incremental blurring approach to model the scatter and geometric response (Bai et al 1998, 2000, Zeng et al 1998, 1999). This approach is an extension of a method that was originally implemented for uniform attenuators by producing an effective scatter source estimation (ESSE) where the activity image is convolved with several convolution kernels to obtain the ESSE (Frey and Tsui 1996, Kadmas et al 1998). The ESSE is estimated by forward projection using a projector that models both attenuation and geometric detector response effects. The method models the non-uniform attenuation effect from the scattering point to the detector, but does not model the non-uniform attenuation from the source to the scattering point. The incremental blurring approach (Bai et al 2000) uses the Klein–Nishina formula to calculate the first-order Compton scatter and an effective scatter source image (ESSI) is obtained using a slice-by-slice blurring model, with the blurring kernels approximated from a transformed Klein–Nishina formula. Scatter projections can be obtained by forward-projecting the ESSI using an incremental blurring projector (Bai et al 1998, Zeng et al 1998), which models non-uniform attenuation and geometric detector response effects.

Most linear algebraic scatter and geometric response compensation methods are image space reconstruction methods. It is assumed that the continuous activity distribution is digitized as a pixel or voxel representation. Ray-driven and pixel-driven projector–backprojectors that model the scatter and geometric response are implemented to form the projection and backprojection images. Using the projection as an example, a pixel-driven projection is performed as the indexing is accomplished sequentially through the pixel array: first by searching the nearest projection bin to the projection of the centre of a given pixel; second by summing to this projection bin the concentration value of the pixel times the point spread function (PSF) of the imaging system. On the contrary, a ray-driven projection cycles through the image array along projection rays driven from the detector. For a given bin, the projection value is computed by accumulating the concentration value of each pixel belonging to the ray times the PSF. In the ray-driven models the geometric response can be approximated by a fan (Tsui et al 1988), or a cone (Zeng et al 1991) of rays emanating from the projection bin with
each ray weighted to match the geometric response of the collimator. The technique we are proposing also uses rays to model the geometric response; however, we have extended the approach to the modelling of the scatter response. Even though most scatter and geometric response compensation methods are image space reconstruction methods, a few publications have proposed projection space reconstruction methods to correct for the effects of attenuation (Gullberg et al 1996) and geometric response (Hsieh et al 1998).

The work here is an extension to 3D of the work of Welch et al (1995). The previous 2D model was far from exact since the origin of detected scatter events cannot be restricted to only a plane but can originate from anywhere throughout the entire volume of the patient’s body. Other models have already been developed that can provide very accurate three-dimensional (3D) scatter compensation in SPECT (Beekman et al 1993, 1996, Ju et al 1995, Frey and Tsui 1996, Frey et al 1993). However, their major drawbacks are that their application is limited to homogeneous media and to a specific acquisition geometry. The method presented in this paper models the distribution of scattered events in the emission projection data by using a variable attenuation map to estimate first-order scatter at each image site by projecting and backprojecting along all possible lines of scatter using a ray-driven projector-backprojector. The 3D geometric response (detector response) is also modelled using the ray-driven projector-backprojector in the work of Zeng et al (1991). The computations in the ray-tracing method are based on the method presented in Gullberg et al (1989). The method has been tested on a numerical simulation and two sets of phantom data, acquired with a two-detector camera equipped with parallel collimators. The results presented are compared with results obtained with various combinations of no correction, attenuation correction, geometric response correction and scatter correction.

2. Theory

For this section, integral equations are used to describe the model used. In section 3, discrete equations will be used, corresponding to the discretization of the equations developed in the present section.

2.1. Attenuation

The photon attenuation from an emission point \( e \) to a detection point \( d \) is often modelled by the following equation:

\[
A(e, d) = \exp \left( - \int_{e}^{d} M(x) \, dx \right)
\]

where \( M \) is the attenuation distribution.

2.2. Three-dimensional Compton scatter model

The detection probability of photons originating from emission point \( e \), scattered at multiple scatter points \( s_i \), and detected at point \( d \), can be modelled as the complete attenuated path from \( e \) to \( d \) (including deviations due to scatter), multiplied by the probability of scatter at each scattering site.

In practice, the method developed must describe as precisely as possible the path of a photon emitted from a given emission point and received by the detection point, including one or several deviations due to scatter. Since at least 80% of the scattered events detected in a \(^{99m}\text{Tc}\) study are the result of a single Compton interaction (Floyd et al 1984), we restrict the model to first-order scatter. Therefore, the detection probability of a photon depends first on
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Figure 1. Modelling of the projection including attenuation, scatter and geometric point response. A photon starting from an emission point \( e \) (or ray \( I \)), scattered at point \( s \), is detected at point \( d \). The different planes represent different possible scattering angles in 3D. The cone of rays near the detection point \( d \) represents the angles of acceptance of the collimator hole.

The attenuation along the path: from source point to scatter point, and from scatter point to detection point (see figure 1).

Theoretically, scatter probability is the combination of the Klein–Nishina formula with the energy-dependent detection probability function of the gamma-camera. Rather than attempt to accurately model the distribution of first-order scatter, we decided to use the product of a simple analytical function to evaluate the scatter probability through a given scattering angle and the attenuation coefficient at the scattering site (Welch et al 1995).

Consequently, the number of photons, emitted from point \( e \), scattered at site \( s \), and detected at point \( d \), is defined by:

\[
P_s(e,s,d) = F(e)A(e,s)/\Psi_1(\phi)M(s)A(s,d)
\]

where \( F \) is the radionuclide distribution and \( \Psi(\phi) \) is the scattering probability function depending on the scattering angle, \( \phi \), defined by the ray from \( e \) to \( s \), and the ray from \( s \) to \( d \) (see figure 1).

This equation can be explained as follows:
(a) A number of photons \( F(e) \) are emitted from \( e \).
(b) These photons undergo an attenuation factor \( A(e,s) \) from the emission source point to the scattering point.
(c) Their scatter probability is expressed by \( \Psi(\phi)M(s) \).
(d) They ultimately undergo a final attenuation factor \( A(s,d) \) from the scattering point to the point of detection.

2.3. Geometric point response

The geometric point response function is the photon fluence distribution on the detector face determined by the geometrical aperture of the collimator hole. In the following, the geometric point response is derived by considering one collimator hole.
For a point source located at a distance $Z$ from the detector plane (see figure 2), an expression for the geometric point response in terms of the autocorrelation of the collimator aperture function $\alpha$ for one collimator hole on the front plane of the collimator can be given by (Tsui and Gullberg 1990, Metz et al 1980):

$$G(r_T) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \alpha(\sigma)\alpha(\sigma + r_T) \, d\sigma$$

(3)

where $r_T$ is a function of $r$, $r_0$ the perpendicular projection of the point source on the detection plane, and $Z$ (see equation (5)). Both $\sigma$ and $r_T$ are on the $\sigma$-plane, which is obtained through a coordinate transformation from the $r$-plane (Tsui and Gullberg 1990).

In this paper, the collimator aperture function $\alpha$ is assumed to be circular. For a radius $R$, the integral in equation (3) can be evaluated by computing the common area of two circles (see figure 2). As described in Zeng et al (1991), equation (3) becomes:

$$G(r_T) = 2 \arccos \left( \frac{r_T}{2R} \right) - \frac{r_T}{2R} \sqrt{1 - \frac{r_T^2}{4R^2}}.$$ 

(4)

It can be shown that for parallel geometry the parameter $r_T$ in equations (3) and (4) is given by (see figure 3):

$$r_T = \frac{L}{Z} (r - r_0)$$

(5)

where $r$ is the polar coordinate of detection point $d$, and $r_0$ is the polar coordinate of the point source projection.
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Figure 3. Computation of $r_T$ for parallel geometry.

In this paper we have focused on parallel geometry, but an analogue model can easily be derived for fan-beam and cone-beam geometry as described in Zeng et al (1991).

Therefore, a projection model of the photons emitted from point $e$ including geometric point response can be derived by:

$$ P_{pr}(e, d) = F(e)G(r_T)A(e, d) $$

where $r_T$ depends on the detection point $d$ (equation (5)). This equation describes the model for the process of acquiring the primary photons.

2.4. Simultaneous modelling of attenuation, scatter and geometric response

Combining equations (1), (2), and (6), a model of the projection process including attenuation, scatter and geometric response can be described by the following equation (see also figure 1):

$$ P_{sc}(e, s, d) = F(e)A(e, s)\Psi(\phi)M(s)A(s, d)G(r_T). $$

For computation purposes equation (7) is now rewritten for a ray $I$ drawn from scattering site $s$. The number of photons emitted from ray $I$ and detected at point $d$ is equal to:

$$ P_{sc}(I, s, d) = \left( \int_I F(x)A(x, s) dx \right) \Psi(\phi)M(s,A(s, d)G(r_T)). $$

The integral describes the number of attenuated photons emitted from points of ray $I$ and scattered at site $s$. The product $\Psi(\phi)M(v)$ describes the scatter probability. The last term corresponds to the projection, including attenuation effect, of the scattered photons onto the detector point $d$.

As an approximation, it is considered that scatter is negligible for scattering angles greater than $\pi/2$. That is to say $\Psi(\phi) = 0$ for $\phi > \pi/2$. Therefore, integrating over the half-space $V/2$ defined by the scattering point and a plane parallel to the detector (see figure 4), we get:

$$ P_{sc}(s, d) = \left( \int_{V/2} \Psi(\phi) dI \int_I F(x)A(x, s) dx \right) M(s)G(r_T)A(s, d) $$

$$ = F_{sc}(s)G(r_T)A(s, d) $$
where $F_{sc}$ can be defined as the scattered radionuclide distribution. Note the similarity between equation (6) and equation (9b). Equation (6) models the projection of the radionuclide distribution, while equation (9b) models the projection of the scattered radionuclide distribution. In other words, in the first case, the acquisition of primary photons is described, while in the latter case the acquisition of scattered photons is described.

3. Methods

3.1. Reconstruction algorithm

Several studies have developed the idea of using scatter correction only in the projector or in a subtraction scheme (Welch and Gullberg 1998). However, the use of a transition matrix that includes all of the corrections for both projection and backprojection steps is theoretically the most appropriate. Therefore, in this study a basic EM algorithm was applied for which an iteration can be written as:

$$
\begin{align*}
    f_{n+1}^e &= \frac{f_n^e}{\sum_d R_{ed}} \sum_d \left( R_{ed} \frac{p_d}{\sum_{e'} R_{d,e'} f_{e'}^n} \right) \\
    \text{where } f &\text{ is the distribution image to be reconstructed, } p \text{ represents the projection data and } R_{ed} \text{ is an element of the transition matrix—including factors for attenuation, scatter and geometric response. In this equation, } n \text{ is the iteration number, } e \text{ corresponds to the voxel index and } d \text{ is the projection bin index.}
\end{align*}
$$

A voxel and a projection bin are said to be linked if a ray emitted from the bin passes through the voxel. The elements $R_{ed}$ are derived by discretizing equations (6) and (7) to give

$$
R_{ed} = \begin{cases} 
    a(e, d) g(r_T) & \text{if } e \text{ and } d \text{ are linked} \\
    a(e, s) \psi(\phi) \mu(s) a(s, d) g(r_T) & \text{otherwise}
\end{cases}
$$

where $a(e, d) = \exp(-\sum_{s \in \text{voxels}} \mu(s))$ is the attenuation factor from voxel $e$ to detection point $d$. All the capital letters corresponding to integral equations (6) and (7) have been replaced by small letters: $\mu$ for $M$, $a$ for $A$, $g$ for $G$, and $\psi$ for $\Psi$. The first case corresponds to projection of primary photons only, while the second case describes the effect of scatter.
3.2. Projector/backprojector

The ideal way to solve the system \( p = Rf \) would be to store the transition matrix \( R \), then use matrix arithmetic to compute equation (10). However, the matrix \( R \) is generally too large to be stored in memory, even if no physical correction is included. The number of non-zero elements is even larger when geometric response and scatter correction are applied, especially when a 3D model is used, because a projection bin is connected to nearly half of the voxels in the image (we suppose that the scatter probability is zero when the scatter angle is greater than 90°). Therefore, all the ray-sums are computed directly for each iteration, both for projection and backprojection, using the algorithm described in (Gullberg et al 1985, 1989).

The algorithm used to compute the projection step is as follows:

loop over projection views
  loop over voxels \( s \)
    loop over rays \( I \)
      compute ray sum (step 1)
      multiply ray-sum by scattering probability (step 2)
      multiply scatter image by attenuation map (step 3)
      add scatter image and primary image
    loop over bin \( d \)
      loop over rays \( I' \)
        compute ray sum (step 4)
        multiply by geometric response factor (step 5)
  After steps 1, 2 and 3, the so-called scatter image \( f_{sc} \), which corresponds to the discretized version of \( F_{sc} \) in equation (9b), is obtained. As noted before, equations (6) and (7) are very similar. This is why the scatter image and primary image are added before projection, rather than projecting them separately.

The backprojection step is very similar to the projection step:

loop over projection views
  loop over bins \( d \)
    loop over rays \( I' \)
      multiply bin value by geometric response factor (step 5)
      compute backprojection ray sum (step 4)
  loop over voxels \( s \)
    loop over rays \( I \)
      multiply pixel value by attenuation coefficient (step 3)
      multiply pixel value by scattering probability (step 2)
      compute backprojected ray sum (step 1)
      add scatter image and primary image

3.3. Estimation of scatter function parameters

One key point of the scatter correction is the use of an analytic function \( \Psi \) (see equation (2)) to model the scatter probability. According to Welch et al (1995), we decided to use a Gaussian function defined as:

\[
\Psi(\phi) = \omega \exp \left( -\frac{(\phi - \phi_0)^2}{w^2} \right)
\]

(12)

where \( \omega \) is the amplitude, \( \phi_0 \) is the position of the centre, and \( w \) is the width of the Gaussian.
To evaluate the parameters of the Gaussian, we used the projections of a point source acquired on a PRISM 2000 camera (Picker International Inc., Cleveland, OH, USA) equipped with parallel collimators. The point source was placed in a large Jaszczak torso phantom (figure 5) (Data Spectrum Corporation, Hillsborough, NC, USA). The phantom was filled with water. No activity was injected in any part of it. The transmission data were acquired using a scanning line source. Transmission and emission data were acquired simultaneously at 120 angles uniformly distributed over 360°. The acquisition time was 30 s per step for the emission data, and 29 s per step for the transmission data. The transmission acquisition time was 1 s less than the emission time in order to ensure that the scanning line source comes to a stop before the detector rotates to the next stop.

Attenuation and activity images were reconstructed using 8 and 30 iterations respectively of the ML-EM (maximum likelihood-expectation maximization) algorithm. The emission image was then thresholded to keep the maximum value, considered to correspond to the location of the point source. Figure 6 presents two slices through the phantom, which show the location of the point source.

3.4. Experiments

3.4.1. Monte Carlo simulation. The PHG release 2.5 Monte Carlo program (Vannoy 1994) was used to simulate projection data for a SPECT scan of the MCAT phantom (Tsui et al 1994). The simulated scan consisted of 120 evenly spaced views over 360° and utilized a primary photon energy of 140 keV. The imaging system simulated a gamma-camera equipped with parallel collimators.
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3.4.2. Jaszczak phantom. An image acquisition experiment was performed using the large Jaszczak torso phantom (see figure 5). For this experiment, a concentration of 66.6 kBq ml$^{-1}$ (1.8 $\mu$Ci ml$^{-1}$) of $^{99m}$Tc was injected in both the myocardial and liver portions of the phantom. Such a concentration is typical of clinical myocardial perfusion studies. No activity was placed in the background.

The acquisitions were made with a two-detector PRISM 2000 camera (Picker International Inc., Cleveland, OH, USA) equipped with parallel collimators to avoid truncation. The acquisition times were 30 s per step for the emission data and 29 s per step for the transmission data. There were 120 projection views, culminating in a 1 h experiment.

3.4.3. Reconstruction parameters. Reconstructions of both the simulated and acquired data were performed using the ML-EM algorithm described in section 3.1, either without any correction or combining attenuation (AC) with geometric response correction (AGC), or geometric response and scatter correction (with a 2D scatter model, AGS2dC, or a 3D model, AGS3dC). For the NC and AC methods, 20 iterations were computed. For the AGC, AGS2dC and AGS3dC methods, 20 iterations were also performed, but the initial image was

![Figure 7. Process of masking images used to compute NSD and contrast.](image)
the attenuation corrected reconstructed image. The goal was to shorten the reconstruction time.

### 3.5. Data analysis

Several image metrics were used to quantitatively evaluate the scatter correction methods:

- The normalized standard deviation (NSD) in the myocardium containing \( N \) voxels was calculated using:
  \[
  \text{NSD} = \frac{1}{\sqrt{N}} \sqrt{\frac{\sum_{i}^{N} (f_m^i - \bar{f}_m)^2}{N-1}}
  \]  
  (13)
  where \( \bar{f}_m \) represents the average pixel value in the myocardium. The NSD provides a measure of uniformity and noise in the myocardium (Beekman et al 1997b).

- The contrast \((C)\) in the myocardium was evaluated using:
  \[
  C = \frac{|\bar{f}_m - \bar{f}_b|}{\bar{f}_m + \bar{f}_b}
  \]  
  (14)
  where \( \bar{f}_b \) is the average value of the ventricular blood pool.

- For a global measure of accuracy of the MCAT (Tsui et al 1994) phantom reconstruction, the sum of squares of differences (SSD) was calculated for each reconstructed image \( f^r \) versus the digital phantom \( f^d \):
  \[
  \text{SSD} = \sum_{i}^{N} (f^r_i - f^d_i)^2.
  \]  
  (15)

- For further quantitative assessment, image profiles through reconstructed images were plotted.

To compute the NSD and the contrast, a masking image was used to estimate the average value in the myocardium and the ventricular blood pool. For the MCAT study, the masking image was the true image itself. For the Jaszczak phantom study, the masking image was heuristically estimated from the reconstructed images (see figure 7).

For all of these computations, the images were first reoriented and then normalized so that the total number of counts in the reconstructed image was equal to the total number of counts in the digitized image for the MCAT experiment and to the total number of counts in the attenuation plus geometric response corrected image for the Jaszczak phantom experiment.

### 4. Results

From the thresholded reconstructed image, a set of projections was generated using the attenuation, 3D scatter and detector response model described in section 2. Figure 8 shows the estimated line spread functions compared with the acquired data, for four different projection views (0, 90, 180 and 270°). From this figure, it can be seen that a good fit between the data and the estimated projections is obtained.

Figure 9 presents selected slices through the reconstructed MCAT simulation, compared with the original image. The images were first reoriented, to allow the visualization of short-axis and long-axis slices. Profiles through these slices, indicated as white lines in figure 9 are presented in figure 10, to compare the different combinations of corrections. In figure 11, profiles of the images reconstructed using the AGS2dC and AGS3dC methods are plotted.
Correction technique for SPECT

Figure 8. Experimental (dotted curve) and estimated (full curve) line spread functions for the large torso phantom calculated for four projection views: (a) 0°, (b) 90°, (c) 180° and (d) 270°.

For the Jaszczak phantom, images of short- and long-axis slices are presented in figure 12. The images were also reoriented. Profiles through these slices are presented in figures 13 and 14.

From all these figures, an incremental increase of image quality with the number of corrections employed can be detected. For the MCAT phantom, the scattered activity around the spine is lowered when scatter correction is used, especially with the 3D scatter model. The geometric response correction allows a thinning of the myocardial wall, compared with the NC and AC methods. Including scatter correction with AGS2dC or AGS3dC seems also to increase the contrast between the left ventricle and the left ventricular chamber. For the Jaszczak phantom, the differences are more difficult to discern because the contrast between the ventricle and the background is modified by the high activity in the liver. However, similar conclusions can be drawn. Using geometric response correction (AGC) leads to thinner ventricular walls, when compared with NC or AC methods. Including scatter correction (AGS2dC or AGS3dC) makes the reconstructed activity in the ventricle more uniform.
Figure 9. MCAT experiment: (left) long-axis slices, (right) short-axis slices. O: original; NC: no correction; AC: attenuation correction; AGC: attenuation and geometric response correction; AGS2dC: attenuation, geometric response and 2D scatter correction; AGS3dC: attenuation, geometric response and 3D scatter correction.

Table 1 shows the results of the quantitative analysis presented in section 3.5. There is no estimation of the SSD for the Jaszczak phantom, since the true distribution is not known. The AGS3dC method provides the best quantitative results.

5. Discussion and conclusion

In this study we have presented a SPECT reconstruction technique which accounts for non-uniform attenuation, scatter and geometric point response of the detector. The course of a photon from its emission point to its detection point on the detector can be summarized as: the photon travels through an inhomogeneous medium (the patient body) where it undergoes
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Figure 10. MCAT experiment: (top) profiles through long-axis slices (as indicated in figure 9), (bottom) profiles through short-axis slices. NC: no correction; AC: attenuation correction; AGC: attenuation and geometric response correction; AGS2dC: attenuation, geometric response and 2D scatter correction; AGS3dC: attenuation, geometric response and 3D scatter correction.

Table 1. Quantitative analysis for the MCAT Monte Carlo simulation and the Jaszczak phantom experiment: computation of contrast (C), normalized standard deviation (NSD) and sum of squares of differences (SSD) for the non-corrected image (NC), attenuation corrected image (AC), attenuation plus geometric response corrected image (AGC), and the attenuation, geometric response plus scatter corrected image, with the 2D scatter model (AGS2dC) or 3D scatter model (AGS3dC).

<table>
<thead>
<tr>
<th></th>
<th>MCAT C</th>
<th>NSD</th>
<th>SSD</th>
<th>Phantom C</th>
<th>NSD</th>
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<td>0.49</td>
<td>2.53</td>
<td>2.89×10^{-5}</td>
<td>0.53</td>
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<tr>
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<td>1.01</td>
<td>2.59×10^{-5}</td>
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<td>0.26</td>
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<tr>
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<td>0.57</td>
<td>2.13×10^{-5}</td>
<td>0.62</td>
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<tr>
<td>AGS2dC</td>
<td>0.64</td>
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<td>2.04×10^{-5}</td>
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<td>0.17</td>
</tr>
<tr>
<td>AGS3dC</td>
<td>0.69</td>
<td>0.41</td>
<td>1.98×10^{-5}</td>
<td>0.83</td>
<td>0.18</td>
</tr>
</tbody>
</table>

several interactions, mostly Compton scatter interactions; after the last interaction, the photon travels to the detector.

The model described in this paper is a sound approximation of the physics of the detection process. First, it takes into account the inhomogeneity of the attenuating media by using the attenuation map reconstructed from the transmission data acquired with the corresponding
Figure 11. MCAT experiment: (top) profiles through long-axis slices (as indicated in figure 9), (bottom) profiles through short-axis slices reconstructed with attenuation, geometric response and 2D scatter correction (AGS2dC) or 3D scatter correction (AGS3dC).

emission data. This allows not only an accurate description of the path of a photon from the emission to the detection point, but also a description of its path before and after scattering has occurred. Second, the scatter model uses an analytical expression to estimate the distribution of the scattered photons. This model incorporates two approximations: it restricts the process to first-order scattering and the attenuation coefficient $M(s)$ should actually be the electron density. However, due to the fit of the scatter probability function to real data, this model also provides a good approximation of the complete distribution of scattered photons as well as other real events such as the increase in the attenuation map values after scatter. This scatter model also allows a good approximation of the tails of the PSF, but not at the peak. The geometric response correction does compensate for this, as can be seen in figure 8. This figure presents the experimental and estimated point spread function for a number of positions. The broad shape of the PSF was well reproduced at all of the positions. The point source experiment shows that the complete model presented in this paper fits the experimental data well.

The algorithm was implemented using a ray-driven technique because it is easier to calculate attenuation factors with the ray-driven technique than it is with the pixel-driven technique. As the ray travels from the detector through the image voxels it is easy to accumulate line integrals of the attenuation coefficients and to calculate the attenuation factor for each pixel (Gullberg et al 1989). However, this results in long reconstruction times—approximately 8 h per iteration for a $64 \times 64 \times 30$ volume on a Sun Ultra Enterprise 3000 with a 300 MHz processor. Several methods can be used to reduce the computation times. First, the ordered subsets-
expectation maximization (OS-EM) algorithm allows a dramatic reduction of the number of iterations required. Second, it is possible to use a scatter subtraction scheme (Welch and Gullberg 1998). This model uses an estimation of the scatter from an attenuation-corrected image only once. The scatter portion is then subtracted from the projection data to reconstruct a scatter-corrected image using only attenuation and geometric point response correction. In this case, the scatter portion of the projection is estimated once, which allows a tremendous reduction in computation time.

From the figures included in this paper, a qualitative analysis can be performed. All of the figures show that the image quality is improved as the number of corrections used is increased. It can be seen from the slices and the profiles in figures 9 and 12 that the combination of attenuation, geometric point response and 3D scatter correction portrays a myocardium with sharper and thinner walls in comparison with the other results.

In this paper, a study was performed to demonstrate the incremental changes that modelling attenuation, scatter and geometric response have on SPECT. The quantitative analysis confirms this qualitative analysis. Compared with the uncorrected reconstructed image, attenuation correction allows better uniformity in the myocardium, as expressed in the decreased NSD
ratio (2.53 versus 1.01 for the MCAT, and 7.61 versus 0.26 for the Jaszczak phantom). This improved uniformity is due to better equalization of the levels of the septal and inferior walls. Combining the geometric response and the scatter model preserves this property. As expected, attenuation does not ameliorate the contrast. This can be explained simply by the fact that both the background and the myocardium values are corrected in the same way when attenuation is taken into account. Compared with attenuation correction, geometric point response correction does not actually improve the uniformity in the myocardium. However, the slightly better NSD value can be explained by the fact that geometric point response correction requires more iterations in order to be efficiently employed in the reconstruction process (Wilson and Tsui 1994). As can be seen in the slices and the profiles, the major influence of correcting for the geometric point response is to thin the walls of the myocardium. Finally, scatter correction has a major impact on contrast quality. This factor is dramatically improved in both the MCAT and the Jaszczak phantom experiments.

The developed model fully incorporates the three-dimensional nature of the problem. It has already been shown that a 3D model outperforms a 2D model (Beekman et al 1996). From the results presented here, it can be seen in figures 11 and 14, and table 1 that the 3D model yields the best quantification of the figures of merit we were interested in. This is caused by the high concentration value in the liver which affects the true value located in the myocardium.
Correlation technique for SPECT

Figure 14. Large Jaszcak torso phantom experiment: (top) profiles through long-axis slices (as indicated in figure 12), (bottom) profiles through short-axis slices reconstructed with attenuation, geometric response and 2D scatter correction (AGS2dC) or 3D scatter correction (AGS3dC).

This paper describes a comprehensive method that accounts for the major degrading effects in SPECT: attenuation, scatter and detector geometric response. We have demonstrated the incremental changes that attenuation correction, scatter correction and geometric response correction have on SPECT. We have also compared the influence of a 3D scatter model with a 2D model, and found the 3D model to be superior. However, a more thorough evaluation is necessary to effectively measure the importance of this factor. Also, we will work to demonstrate the importance of modelling non-uniform attenuation between the source and scatter point, as opposed to methods that assume a uniform attenuator (Frey and Tsui 1996).

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