h(w, z) for this system is
\[ h(w, z) = 1 - 1.2z + 0.3z^2 - 1.5w + 1.8wz - 0.75wz^2 + 0.6w^2 - 0.72w^2z + 0.29wz^2z^2. \]
Here, \( m = n = 2 \), \( k_{11} = k_{21} = 0 \), \( k_{12} = 0.3 \), and \( k_{22} = -0.11 \). Hence, checking both first and second statements, it follows that

\[ A. \]

Here, \( m = n = 2 \), \( k_{11} = k_{21} = 0 \), \( k_{12} = 0.3 \), and \( k_{22} = -0.11 \). Hence, checking both first and second statements, it follows that

\[ \hat{p}_{11}(\lambda) = \lambda^2 - (1.5)\lambda + 0.6 = 0 \]
\[ \Rightarrow \lambda_1 = 1.137 \quad \text{and} \quad \lambda_2 = 0.3627 \]
\[ \hat{p}_{22}(\lambda, 1) = \lambda^2 - (1.2)\lambda + 1.6 = 0 \]
\[ \Rightarrow \lambda_1 = 2 \quad \text{and} \quad \lambda_2 = -0.8 \]
\[ \hat{p}_{22}(\lambda, -1) = \lambda^2 - (1.2)\lambda + 0.4322 \]
\[ = 0 \quad \Rightarrow |\lambda_{1,2}| = 0.6 + j0.2687. \]

Hence the system is unstable, which is in agreement with the results reported in [3].

**Example 3:** Consider the transfer function whose denominator \( h(w, z) \) is [12]:
\[ h(w, z) = 1 - w - 0.7z + 0.67wz + 0.25w^2 - 0.16wz^2. \]
Here, \( m = 2, n = 1, k_{12} = k_{22} = 0, k_{11} = 0.03, \) and \( k_{21} = -0.015 \). Hence, checking the criteria of the first statement, we have
\[ \hat{p}_{11}(\lambda) = \lambda^2 - (-1)\lambda + 0.25 = 0 \quad \Rightarrow \lambda_1 = \lambda_2 = 0.25 \]
\[ \hat{p}_{22}(\lambda, 1) = \lambda + (-0.7 - 0.03 - 0.015)/(0.25) = 0 \]
\[ \Rightarrow \lambda = 0.76 \]
\[ \hat{p}_{22}(\lambda, -1) = \lambda + (-0.7 - 0.03 - 0.015)/(2.25) = 0 \]
\[ \Rightarrow \lambda = 0.68. \]
Similarly for the second statement:
\[ \hat{p}_{22}(\lambda) = \lambda + (-0.7) = 0 \quad \Rightarrow \lambda = 0.7 \]
\[ \hat{p}_{11}(\lambda, 1) = \lambda^2 - 1.1\lambda + 0.3 = 0 \]
\[ \Rightarrow \lambda_1 = 0.6 \quad \text{and} \quad \lambda_2 = 0.5 \]
\[ \hat{p}_{22}(\lambda, -1) = \lambda^2 - 1.01764\lambda + 0.24117 = 0 \]
\[ \Rightarrow \lambda_1 = 0.64 \quad \text{and} \quad \lambda_2 = 0.37. \]
Hence, checking both first and second statements, it follows that the system is stable.

Note that in all the above examples we applied our procedure assuming the variables \( w \) and \( z \) as \( w^{-1} \) and \( z^{-1} \), respectively.

**IV. Conclusions**

A simple procedure is presented that facilitates the application of the stability state-space criteria presented in [6]. This procedure is based on the results reported in [7], wherein an algorithm is given that yields a simple form for the system matrix \( A \). This form of \( A \) simplifies the derivation of the 1-D polynomials required for checking the stability of a 2-D system.

**References**


Comments on "Adaptive Algorithms with an Automatic Gain Control Feature" by SHLOMO KARNI AND GENGSHENG ZENG

*Abstract*—It is shown that the new LMS algorithm introduced in the above paper is not properly devised and that it may make the algorithm itself divergent or "sleep" forever.

In the above paper, Shan and Kailath proposed a new LMS algorithm as follows:

\[ W_{k+1} = W_k + g_k e_k X_k X_k^T \]  
\[ s_{k+1} = \alpha s_k \]  
\[ \hat{p}_k = \lambda \hat{p}_{k-1} + (1 - \lambda) e_k e_k^T \]  
\[ \hat{e}_k = \frac{1}{M} \sum_{i=1}^{M} e_i^T. \]

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$M$ is the dimension, and $x_i^j$ is the $j$th component of the $k$th input vector $X$.

1) It is well known that a necessary condition for the algorithm (1) to converge is

$$g_k > 0$$

i.e., $W_{k+1}$ should move in the opposite direction of the gradient of the performance surface. However, the $g_k$ given in (2) can be positive or negative.

2) In order to guarantee the stability of the algorithm, the convergence factor $q_k$ should be upper bounded, that is,

$$0 < q_k < g$$

(6)

Here $g$ is a positive constant, determined by the statistics of $(X)$. For some large errors $\epsilon$, the proposed $g_{k+1}$ in (2) may exceed the upper bound $g$.

3) By the orthogonality principle, the algorithm converges if and only if

$$E[eX] = 0 \quad \text{(Vector equation).}$$

(7)

This condition is much stronger than the one the authors used:

$$E[e\tilde{x}] = 0 \quad \text{(Scalar equation).}$$

(8)

In fact, (7) implies (8), but not vice versa. If the dimension of $X$ is large, $\tilde{x}$ will be very close to zero, and the algorithm will "sleep" forever.

Comments on "Comments on 'Adaptive Algorithms with Automatic Gain Control Feature'"

T. J. Shan and T. Kailath

We thank Dr. Karni and Dr. Zeng for their interest in our paper.

The first comment that $g_k$ should be $> 0$ is very true. Actually our equation (2) should have read

$$g_{k+1} = a|\hat{p}_k|$$

which of course is $> 0$. The conference version of our paper [1] did have the absolute magnitude signs for $\hat{p}_k$, and we regret not having caught this error.

The second comment that $g_k$ should be upperbounded is also true. It is a universal assumption for such algorithms, which is perhaps why we did not explicitly mention it.

Comment 3) is the most significant one and means that the algorithm may tend not to work for high-dimensional vector $X$. However, a more definite statement will need accurate convergence analysis, which is not at hand.

References


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