Iterative and analytical reconstruction algorithms for varying-focal-length cone-beam projections

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Iterative and analytical reconstruction algorithms for varying-focal-length cone-beam projections

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Abstract. In single photon emission computed tomography (SPECT), a varying-focal-length cone-beam collimator can be used to reduce the truncation problem and to maintain sensitivity when imaging the organ of interest. The collimator is constructed so that the collimator holes focus to a circular symmetric, spatially varying, focal point function. The focal length increases radially from the shortest focal length at the centre to the longest focal length at the periphery of the collimator. This paper describes a 3D backprojection–filtering (BF) algorithm for this varying-focal-length cone-beam geometry. The proposed algorithm is compared to an iterative ML–EM (maximum likelihood–expectation maximization) algorithm. The 3D Hoffman brain phantom and Defrise phantom are used in computer simulations. Since the maximum tilt angle of the projection rays is small for most realistic imaging geometries, the proposed algorithm provides a good approximation. When a circular orbit is used, the BF algorithm gives an exact reconstruction of the central slice.

1. Introduction

In single photon emission computed tomography (SPECT), a cone-beam collimator is used with a large-field-of-view rotating gamma camera to improve sensitivity for imaging smaller organs such as the brain (Jaszczak et al 1986, 1988) and heart (Gullberg et al 1991). The magnification of the cone-beam collimation can cause significant truncation of background tissue. A varying-focal-length cone-beam collimator can be used to maintain high sensitivity when imaging the organ of interest with minimal truncation of the body occurring at the edge of the field of view. The focal lengths increase radially and symmetrically from a minimum at the centre to a maximum at the edge of the collimator. Employing this configuration, the central region of interest is imaged with short focal lengths and high sensitivity. In using this method, tissues close to the edge of the patient’s body are imaged with nearly parallel rays, thereby avoiding or reducing projection truncation, which is an inherent problem for convergent beam imaging.

Hsieh (1989a) first proposed the concept of a variable-focal-length cone-beam collimator. This concept (figure 1) was also presented by Jaszczak at the First Fully 3D Image Reconstruction Meeting (Jaszczak et al 1992a). In a later abstract he proposed the use of an iterative reconstruction algorithm to reconstruct data collected using this geometry (Jaszczak et al 1992b). For a special kind of varying-focal-length cone-beam collimator,
Guillemaud and Grangeat (1994) proposed a rebinning method that converted the varying-focal-length projections into cone-beam projections that could be used for reconstruction using Grangeat’s cone-beam reconstruction algorithm (Grangeat 1987).

The 3D varying-focal-length cone-beam work in this paper is based upon work that has been performed using a 2D varying-focal-length fan-beam collimator geometry. The idea of this type of developing collimator was originally proposed by Hsieh (1989b). Recently, a number of algorithms have been developed for reconstructing varying-focal-length fan-beam projections. We conjectured in 1993 that no convolution–backprojection existed for this geometry (Zeng et al. 1993), which led us to develop a summed convolution–backprojection algorithm that convolved the varying-focal-length fan-beam projection data with a series of kernels, then backprojected the sum of the convolved projections. This algorithm involved calculating a finite approximation to an infinite series of orthogonal Chebyshev polynomials. Later, Cao and Tsui (1994) published a paper describing a filtered backprojection algorithm with a spatially varying filter that could not be implemented as a convolution. We, however, took a different approach by developing an exact backprojection–filtering algorithm to reconstruct varying-focal-length fan-beam projections (Zeng and Gullberg 1994). The algorithm first backprojected the projection, then performed a two-dimensional shift-invariant filtering.

In this paper, the 2D varying-focal-length fan-beam backprojection–filtering algorithm is extended to obtain a 3D cone-beam version of the algorithm for a circular orbit and a circular sinusoidal orbit scanning geometry. We must point out that in 2D varying-focal-length fan-beam geometries the projection data set is complete when the camera rotates in a full circle. For 3D varying-focal-length cone-beam geometries, the projection data set is not complete when the camera rotates only one full circle (Palamodov 1991). As a result, one can only obtain an approximate algorithm similar to the Feldkamp algorithm for the fixed-focal-length cone-beam geometry (Feldkamp et al. 1984). In the technique described in this paper the gamma camera rotates around the patient in a circular orbit, or in a circular

![Figure 1. Illustration of a varying-focal-length cone-beam geometry, where $D(s)$ is the focal-length function and $s$ is the distance from the detection point to the centre.](image)
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It is not known at this time whether a circular sinusoidal orbit provides a complete projection data set.

In this paper, the analytical backprojection–filtering algorithm to reconstruct varying-focal-length cone-beam projections is described. Simulations are then presented that compare results of the proposed algorithm with results reconstructed using an iterative ML–EM algorithm. The iterative ML–EM algorithm uses a projector/backprojector with a line-length weighting model. Results of computer simulations performed with both noiseless and noisy projection data are shown. Effects of attenuation, system point response function and scatter are not considered in this paper.

2. Methods

2.1. Focal-length function

Each hole on the detector has its own focal length, $D(s)$, where $s$ is the radial distance from the hole to the centre of the detector. We assume that $D(s)$ is a monotonically increasing function of $s$, for example, $D(s) = D_0 + ks^2$, where $D_0$ is the minimum focal length at the centre of the collimator, and $k$ is a positive constant.

<table>
<thead>
<tr>
<th>ROI diameter (cm)</th>
<th>Parallel $D(s) \equiv \infty$</th>
<th>Fan $D(s) \equiv 63$ cm</th>
<th>Varying cone $D(s) = 63 + 0.22s^2$ cm</th>
<th>Cone $D(s) \equiv 63$ cm</th>
</tr>
</thead>
<tbody>
<tr>
<td>21</td>
<td>1.0a</td>
<td>1.80a</td>
<td>1.54a</td>
<td>3.34a</td>
</tr>
<tr>
<td>14</td>
<td>1.0b</td>
<td>1.77b</td>
<td>1.76b</td>
<td>3.15b</td>
</tr>
<tr>
<td>7</td>
<td>1.0c</td>
<td>1.75c</td>
<td>2.12c</td>
<td>3.08c</td>
</tr>
</tbody>
</table>

The varying-focal-length cone-beam collimator has a higher degree of sensitivity than a parallel collimator but not as high a degree of sensitivity as a fixed-focal-length cone-beam collimator. To illustrate this point, table 1 compares sensitivity studies between parallel, fan, varying-focal-length cone and fixed-focal-length cone geometries. In the comparison studies, the varying-focal-length function was $D(s) = D_0 + ks^2$ with $D_0 = 63$ cm and $k = 0.219$ cm$^{-1}$. Both the cone-beam and fan-beam focal lengths were $D \equiv D_0 = 63$ cm. The detector matrix was a $64 \times 64$ array of pixels of 0.7 cm for all collimators. In all cases, the detector was 22.4 cm (32 pixels) from the axis of rotation. Three different spherical objects with diameters of 21, 14 and 7 cm were studied separately. The geometry for the central transaxial section is shown in figure 2.

The parallel, fixed-focal-length cone-beam and varying-focal-length cone-beam geometries are compared in figure 2 and table 1. Note that the varying-focal-length cone-beam geometry has almost the same sensitivity for the central region of interest as the fan-beam geometry. There was no truncation in the sensitivity studies. However, in practice, the patient projections are truncated in fixed-focal-length fan-beam and cone-beam studies. The varying-focal-length geometry has little or no truncation of the object as in the case of a parallel imaging geometry.
2.2. Reconstruction algorithms

Previously we proposed (Zeng and Gullberg 1994) that the image can be reconstructed exactly, from varying-focal-length fan-beam data, by first backprojecting the projection data then filtering the backprojected image with a 2D ramp filter. We use the same technique for the 3D varying-focal-length cone-beam geometry. First, a 3D voxel-driven backprojector was used to backproject the projection data along the varying-focal-length cone-beam geometry, then a 2D ramp filter was applied to each transaxial slice, that was orthogonal to the detector. The 2D filtering was performed in the frequency domain and the image array was zero padded before performing the Fourier transformation. Therefore, our proposed analytical algorithm for the varying-focal-length cone-beam geometry consists of 3D backprojection followed by slice-by-slice 2D filtering. We call the resulting algorithm the backprojection–filtering (BF) algorithm. In the algorithm, we assume that line integrals are obtained before reconstruction. If this is not the case, a preprocessing procedure is required. For example, in the case of flat detector acquisition with uniform sampling, the measured projections must be divided by a cosine function of the tilt angle. The tilt angle is the angle between the projection ray and a line normal to the detector.

An iterative ML–EM algorithm is used to compare results obtained with the backprojection–filtering algorithm. For the varying-focal-length cone-beam geometry, the ML–EM algorithm is almost identical to the one used for the fixed-focal-length cone-beam geometry (Gullberg et al 1989). A 3D, ray-driven, line-length-weighted projector–backprojector pair is used in the algorithm to calculate the projection and backprojection operations.

Theoretically, if continuous 360° measurements are available, the projections are sufficient for an exact reconstruction of the central slice. Exact reconstruction of the central slices is also possible for discrete measurements, provided the angular and linear sampling intervals satisfy the Nyquest sampling principle. However, due to a number of practical factors such as the finite number of projection views, the truncated backprojection
region, the approximated projection/backprojection models and the discrete representation of a continuous function, the reconstruction of the central slice is very close, but not truly exact. If a non-planar orbit (e.g., a circular sinusoidal orbit) is used, we do not know yet whether the projections are complete, nor do we know if theoretically any region of the image can be reconstructed exactly.

In our analytical BF algorithm, the 2D ramp filtering is performed in the planes transaxial to the axis of rotation. Thus, blurring in the axial direction, caused by the backprojection step, is disregarded by the reconstruction algorithm. If the backprojection–filtering algorithm is applied to a circular detector orbit, all of the off-central-plane slices are only approximately reconstructed. The approximation errors are proportional to the tilt angles of the projection rays. For the example given in table 2 the maximum tilt angle of the projection rays in the varying-focal-length cone-beam geometry is small and less than 8°.

### Table 2. Maximum tilt angle $\max_i (\tan^{-1} s/D(s))$.

<table>
<thead>
<tr>
<th>Parallel</th>
<th>Fan</th>
<th>Varying cone</th>
<th>Cone</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D(s) \equiv \infty$</td>
<td>$D(s) \equiv 63$ cm</td>
<td>$D(s) = 63 + 0.22s^2$ cm</td>
<td>$D(s) \equiv 63$ cm</td>
</tr>
<tr>
<td>0°</td>
<td>0°</td>
<td>7.67°</td>
<td>19.57°</td>
</tr>
<tr>
<td>at $s = 16.96$ cm</td>
<td>at edge, i.e., $s = 22.4$ cm</td>
<td>(s = 24.3 pixels)</td>
<td>(s = 32 pixels)</td>
</tr>
</tbody>
</table>

![Figure 3](image)

**Figure 3.** Illustration of a 3D Defrise phantom that consists of five identical discs, each with a radius of 20 pixels (14 cm) and thickness of 6 pixels (4.2 cm). The gap between discs is 3 pixels (2.1 cm).

### 2.3. Computer simulations

A 3D Hoffman brain phantom, and a 3D Defrise phantom, were used in computer simulations. The 3D Defrise phantom is shown in figure 3. Projection data were simulated using the focal-length function given in section 2.1. The projection data were generated by a line-length weighted projector on a $128 \times 128$ array. The projection array was then
Figure 4. Backprojection–filtering reconstructions of the 3D Hoffman brain phantom, using varying-focal-length cone-beam geometry and parallel-beam geometry. Both circular orbit and circular sinusoidal orbit projections are used for the varying-focal-length cone-beam geometry. The profile is drawn along the axis of rotation. The mean square error is for the whole image.

collapsed to a 64 × 64 array by summing up neighbouring pixels. In noisy data studies, the projections were randomized according to Poisson distribution.

There were 120 projection views simulated over 360°. Both circular orbit 1 = \((R \cos \theta, R \sin \theta, 0)^T\)
and circular sinusoidal orbits

orbit 2 = \(\left( R \cos \theta, R \sin \theta, \sin \left( \frac{60}{\pi} \theta \right) \right)^T\)

for 0 ≤ θ < 2π were used in the computer simulations. Here, R = 22.4 cm (or 32 pixels) was the distance from the detector to the rotation axis. For both orbits, the focal line was always perpendicular to the axis of rotation and the camera did not tilt.

Images were reconstructed in a 64 × 64 × 64 array of voxels of 0.7 cm; the image voxel size was equal to the detector pixel size. In performing the iterative reconstructions of the Hoffman brain phantom 21 iterations were used and in performing iterative reconstructions of the Defrise phantom 13 iterations were used. The number of iterations was determined by the minimum point of the curve of mean square error against the iteration number. In order to have accurate images as a standard, parallel-beam data were also used for analytical and iterative reconstructions. For parallel-beam geometry, the number of iterations for the 3D Hoffman phantom was 19 and the number of iterations for the Defrise phantom was 12.
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3. Results

Figure 4 illustrates the varying-focal-length cone-beam backprojection-filtering (BF) reconstructions (central transverse, sagittal and coronal sections) of images obtained from imaging the Hoffman brain phantom with a circular orbit and a circular sinusoidal orbit. Reconstructions with both noiseless data and noisy data are included. The parallel-beam BF reconstructions are also shown in figure 4. Figure 5 shows the iterative ML–EM reconstructions of the same set of studies. The BF and EM reconstructions are very similar, which suggests that the analytical BF algorithm performs nearly as well as the EM algorithm, with the exception that EM reconstructions are less noisy than the BF reconstructions when noisy data are used. One might consider that the iterative ML–EM algorithm provides the best possible reconstruction for a given set of projection data.

In order to demonstrate some reconstruction artefacts, reconstructions of the Defrise phantom are shown in figures 6 and 7. Figure 6 shows the BF algorithm results and figure 7 displays the EM algorithm results. The noiseless studies (see profiles) reveal that the slice-to-slice cross-talk artefacts are more severe in the BF reconstruction than in the EM reconstruction. It is observed that the central disc was accurately reconstructed with the circular orbit. The reconstruction from the sinusoidal orbit data suffered more severe

<table>
<thead>
<tr>
<th></th>
<th>Central Transverse</th>
<th>Central Coronal</th>
<th>Central Sagittal</th>
<th>Profile Through Central Sagittal</th>
<th>Mean Square Error</th>
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<td><img src="image23" alt="Images" /></td>
<td><img src="image24" alt="Images" /></td>
<td>0.0263</td>
</tr>
</tbody>
</table>

Figure 5. Iterative ML–EM reconstructions of the 3D Hoffman brain phantom, using varying-focal-length cone-beam geometry (21 iterations) and parallel-beam geometry (19 iterations). Both circular orbit and circular sinusoidal orbit projections are used for the varying focal-length cone-beam geometry. The profile is drawn along the axis of rotation. The mean square error is for the whole image.
Figure 6. Backprojection–filtering reconstructions of the 3D Defrise phantom, using varying-focal-length cone-beam geometry and parallel-beam geometry. Both circular orbit and circular sinusoidal orbit projections are used for the varying-focal-length cone-beam geometry. The profile is drawn along the axis of rotation. The mean square error is for the whole image. The normalized standard deviation is for a disc at the central slice only.

cross-talk artefacts between slices especially when the analytical BF algorithm was used. The iterative EM algorithm gave much better results. This is because that when a sinusoidal orbit was used the analytical BF algorithm was an approximation everywhere, and none of the slices were exactly reconstructed.

The mean square error (MSE), which is defined as

$$\text{MSE} = \frac{1}{\text{No. of voxels}} \sum_{i=1}^{\text{No. of voxels}} \left[ \text{image}_{\text{reconstruct}}(i) - \text{image}_{\text{true}}(i) \right]^2$$

was calculated for all reconstructions. The MSE values indicate that the BF reconstructions are not as accurate as the EM reconstructions. For the central slice of the Defrise phantom images, the normalized standard deviation (NSD), which is the standard deviation divided by the mean, was also calculated to measure the noise level. The NSD values indicate that the BF reconstructions are noisier than the EM reconstructions. This is due partially to the fact that no regularization was applied in the BF algorithm and that the EM algorithm was stopped at its minimum MSE value. Parallel-beam reconstructions are shown to be a little noisier than the reconstructions with varying-focal-length cone-beam reconstructions with noisy data. This is due to the fact that the parallel geometry has a lower sensitivity than the varying-focal-length cone-beam geometry.
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Figure 7. Iterative ML–EM reconstructions of the 3D Defrise phantom, using varying-focal-length cone-beam geometry (13 iterations) and parallel-beam geometry (12 iterations). Both circular orbit and circular sinusoidal orbit projections are used for the varying-focal-length cone-beam geometry. The profile is drawn along the axis of rotation. The mean square error is for the whole image. The normalized standard deviation is for a disc at the central slice only.

4. Discussion

The varying-focal-length cone-beam imaging geometry offers high image sensitivity at the region of interest while keeping the whole object within the field of view. An efficient analytical backprojection–filtering (BF) algorithm is proposed for this imaging geometry. Since the maximum tilt angle of the projection rays is small for most realistic imaging geometries, the algorithm provides a good approximation. When a circular orbit is used, the BF algorithm gives an exact reconstruction of the central slice.

In 3D Hoffman brain phantom computer simulations, the BF algorithm produces nearly the same reconstructions as those obtained with the EM algorithm, without any visible artefacts present. For projection data corrupted with Poisson noise, the BF reconstructions are noisier than the EM reconstructions, which is a common observation and is due partially to the stopping rule of the EM algorithm.

Parallel-beam geometry has many advantages over the proposed varying-focal-length collimation. For example, a complete set of projection data can be obtained with a parallel geometry with a circular scanning orbit, and the parallel geometry has exact filtered backprojection (FBP) and backprojection–filtering (BF) reconstruction algorithms. On the other hand, the varying-focal-length collimation has a higher sensitivity at the centre of the
region of view, resulting in a higher signal-to-noise ratio at the central region (see table 1). For many clinical applications, a structure similar to the Defrise phantom would be unusual and the data-insufficiency artefacts would not be observable, especially in the central region (see results of Hoffman phantom studies). Therefore, the varying-focal-length cone-beam collimation can be applied in nuclear medicine (Hsieh 1989a).

Computer simulations with the 3D Defrise phantom reveal some artefacts that are inherent for the incomplete data measurement and approximate reconstruction algorithm. The EM reconstruction with noiseless data can be thought of as providing the best possible reconstruction for a given set of projection data. It is also observed that the discs are not very well separated, even in EM reconstructions. This is mostly due to the inherent problem of the incomplete data caused by the collimator and scanning geometry. The disc separation problem is more severe for the BF algorithm than for the EM algorithm. This is because the BF algorithm has cross-slice backprojection, but does not have deblurring filtering between slices.

Due to the fact that 2D ramp filtering is performed in the planes transaxial to the axis of rotation in the BF algorithm, it is required that the detector remains parallel to the axis of rotation and that the scanning orbit be isotropic. If the scanning orbit is not isotropic, for instance if the circle-and-line orbit is used, a special weighting function must be employed in the backprojection step. This will be a subject of future investigation. Further investigation of the data sufficiency conditions for the varying-focal-length cone-beam imaging geometry also needs to be performed in order to develop an exact reconstruction algorithm.

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References

Hsieh J 1989a Scintillation camera and three-dimensional multifocal collimator used therewith United States Patent 4820924
———1989b Scintillation camera and multifocal fan-beam collimator used therein United States Patent 4823017
Varying-focal-length cone-beam projections