Lecture 12
Carrier Phase Synchronization
Digital Communication System

- Information (sound, video, text, data, ...)
- Transducer & A/D Converter
- Source Encoder
- Channel Encoder
- Modulator
- Tx RF System
- Channel
- Rx RF System
- Demodulator
- Channel Decoder
- Source Decoder
- D/A Converter and/or output transducer
- Output Signal
Discrete-time QAM Modulator

Direct Digital Synthesizer

Frequency of interest

Basis Function $\phi_0(t)$

Basis Function $\phi_1(t)$

Example: 16-QAM Constellation

Note that the baseband signal $s(t)$ may be up-converted (multiplied with carrier signal) to higher frequency (e.g., 900 MHz, 2.4 GHz, 5 GHz, etc) in **super-heterodyning**
Discrete-time QAM Demodulator

We will assume that in the super-heterodyne receiver, the high-frequency carrier signal may have been down-converted (with a separate PLL) and we have the baseband signal $r(t)$. $x(kT_s)$ and $y(kT_s)$ contain the original constellation point + noise: Eq. 5.91
Problem: Carrier Phase Offset

Uncompensated carrier phase offset can cause,

1) Counterclockwise (CCW) rotation of $\theta^\circ$

1) Symbols to lie in the wrong Decision Region

This can happen regardless of symbol timing synchronization and absence of noise

How to compensate?

Carrier Phase Synchronization: Estimating the phase of the carrier (at $n=1^{st}$ sample or $t=0^{th}$ second)
Carrier Phase Synchronization

- Typically, carrier signal is received and down-converted to the baseband signal: *phase and frequency of the received signal are unknown*
- For QAM signals, *the received signal has 90 deg. phase shifts*
- PLL designed to track simple sinusoid can not lock

Discrete-time QAM Receiver with Intermediate Frequency (IF) Sampling

- CCW Rotation comes from multiplying the oscillator output with the received I&Q signals
- ADC
- $r(t) \rightarrow r(nT)$
- $I(nT)$
- $x(nT)$
- $x(kT_s)$
- $n = k \frac{T_s}{T}$
- $y(nT)$
- $y(kT_s)$
- $n = k \frac{T_s}{T}$
- $\cos(\Omega_0 n + \hat{\theta})$
- $-\sin(\Omega_0 n + \hat{\theta})$
- $Q(nT)$
- $\text{matched filter}$
- $\text{matched filter}$
Carrier Phase Synchronization: Approach 1

- Carrier phase synchronization will remove the phase shifts and track the remaining phase.
- This task is done by designing a proper phase detector.

**Carrier Phase Synchronization with Phase Adjusted Quadrature Sinusoids**

1) **Eq. 7.7**
2) Compute Phase Error generates error signal using the received symbols $x$ & $y$ and projected symbols $\hat{a}_0$ & $\hat{a}_1$
3) PLL locks when $\hat{\theta} = \theta$

**Problem:** multi-rate system

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1 sample/symbol

$N (= T_s / T)$ samples/symbol
Carrier Phase Synchronization: Approach 2

- First, the sinusoids are down-converted with fixed-freq. oscillators
- Phase compensation is done by Counter Clock Wise (CCW) rotation block
- This is called Decision-directed PLL (uses symbol estimates to compute phase error)
Carrier Phase Synchronization for QPSK
Carrier Phase Synchronization for QPSK

- Eq. 7.17: phase detector output = the error signal for \( k \)th symbol, \( e(k) \)
- Eq. 7.18: Symbol estimates (decision block)

**Heuristic Phase Detector**

![Phase Detector Diagram](image)
**Problem: π/2 Phase Ambiguity**

**Q: Where is the stable lock point?**

**A:** Usually, the lock point is when PLL locks at $\theta_e = 0$. Here the stable lock points are $\theta_e = -\pi/2 \ (\text{-90}^\circ \text{ or 270}^\circ), \ 0 \ (\text{0}^\circ), \ \pi/2 \ (\text{90}^\circ), \ \pi \ (\text{180}^\circ)$

The above figure shows that the QPSK carrier phase PLL with heuristic phase detector can lock onto the carrier at 4 possible points: $0^\circ \ (\text{true}), \ +/- \ 90^\circ \ , \text{ or } 180^\circ \text{ out of phase}$
Carrier Phase Synchronization for QPSK

- **Eq. 7.26**: phase detector output = the error signal for $k^{\text{th}}$ symbol, $e(k)$
- **Eq. 7.27**: Symbol estimates (decision block)

**Max. Likelihood Phase Detector**

![Diagram of a maximum likelihood phase detector for QPSK]

1. **Sampled band-pass signal**: Input to the system.
2. **Matched filter**: Processes the input signal.
3. **Free-running oscillator**: Generates the reference signal.
4. **CCW rotation**: Corrects the phase error.
5. **Decision block**: Outputs the symbol estimates $\hat{a}_0(k)$ and $\hat{a}_1(k)$.
6. **Loop filter**: Filters the error signal $e(k)$.

The diagram illustrates the process of carrier phase synchronization, including the use of matched filters, a free-running oscillator, and a decision block to estimate the symbols and correct the phase error.
Problem: $\pi/2$ Phase Ambiguity

Again, the stable lock points are $\theta_e = -\pi/2, 0, \pi/2, \pi$.

$K = G_a/T$; where $G_a$ = amplitude gain, losses through antennas, channel, amps, mixers, filters, and other RF components

$T$= sampling time of the received signal

$A$= symbol amplitude

The above figure shows that the QPSK carrier phase PLL with max. likelihood phase detector can lock onto the carrier at 4 possible points: $0^\circ$ (true), +/- $90^\circ$, or $180^\circ$ out of phase.
Design Example

Section 7.2.3
Find the loop constants using Eq. C.61
Carrier Phase Synchronization for MQAM
Carrier Phase Synchronization for MQAM

- Similar architecture as QPSK: decision block changes
- Eq. 7.55: phase detector output = the error signal for \( k^{th} \) symbol, \( e(k) \)
- Eq. 7.57-58: Symbol estimates (decision block)

**Fig. 7.1.2**

The diagram illustrates the carrier phase synchronization process for MQAM. The key components include:

- Sampled band-pass signal
- Matched filter
- Free-running oscillator
- Carrier Phase Offset Estimate, \( \hat{\theta} \)
- DDS
- CCW rotation
- Decision block
- Loop filter
- Compute Phase Error
- Timing synchronization

The process involves the following steps:

1. The sampled band-pass signal is filtered and multiplied by the carrier signal to generate \( x(nT) \) and \( y(nT) \).
2. The phase detector outputs the error signal for each symbol, \( e(k) \), which is used to adjust the phase of the carrier signal.
3. Symbol estimates are obtained through the decision block.

The diagram highlights the synchronization process with a focus on the phase detection and correction mechanisms.
Error Function for MQAM

- Multiple stable lock points

8PSK

16QAM
Phase Ambiguity Resolution: Unique Word
How to resolve the phase ambiguity?

- **Unique Word**: Commonly used in wired and wireless communication (also known as *syncword*).

- Insert pattern of known symbols or Unique Word (UW) of 8-bits in the bit stream in Tx.

- At the receiver, the carrier phase lock is obtained (the Tx will repeat the above packet ~ 4 times to allow the PLL transient time to pass).

- Next, the detector compares the estimated symbols and UW to figure out the phase ambiguity.
Unique Word for QPSK

Total of 4 Unique Word Flags

Correct by rotating the symbol estimates
Or
Use a different bit-to-symbol map

Symbol Estimates

\( \hat{a}_0(k) \)

\( \hat{a}_1(k) \)

Phase ambiguity correction

Select phase ambiguity

Find UW(0°)

Find UW(90°)

Find UW(180°)

Find UW(270°)
Unique Word (UW) for QPSK

Example 1

Bit-to-Symbol Map

<table>
<thead>
<tr>
<th>Symbol Index</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\hat{a}_0(k)$</td>
<td>-1</td>
<td>+1</td>
<td>+1</td>
<td>-1</td>
<td>+1</td>
<td>+1</td>
<td>-1</td>
<td>-1</td>
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<td>-1</td>
<td>+1</td>
<td>+1</td>
<td>-1</td>
<td>+1</td>
<td>+1</td>
<td>-1</td>
</tr>
</tbody>
</table>

Received UW:
00 10 11 01

Transmitted UW:
1 0 1 1 0 1 0 0
+1 -1 +1 +1 -1 +1 -1

Q: Which data bits were transmitted?
A: First we need to find out the phase rotation based on the UW.

Transmitted UW: 10 11 01 00
Received UW: 00 10 11 01
Unique Word (UW) for QPSK

Example 1 (cont....)

Transmitted UW
10 11 01 00

Received UW
00 10 11 01

Q: Which one of the following rotations happened?

rotate 0°

Q: Which data bits were transmitted?

rotate CW 90°

rotate CCW 180°

rotate CCW 270°

or CW 90°
**Unique Word (UW) for QPSK**

**Example 2**

- **Bit-to-Symbol Map**
  - Q (out of phase)
  - I (in phase)

- **Transmitted UW**
  - 1 0 1 1 0 1 0 0
  - +1 -1 +1 +1 -1 +1 -1 -1

- **Received UW**
  - 01 00 10 11

- **Received Data**
  - 01 00 10 11

**Symbol Index**

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- **Received Bits**
  - 01 00 10 11

**Q:** Which data bits were transmitted?

**A:** First we need to find out the phase rotation based on the UW.

**Transmitted UW:** 10 11 01 00

**Received UW:** 01 00 10 11
Unique Word (UW) for QPSK

Example 2 (cont. . .)

Transmitted UW
10 11 01 00

Received UW
01 00 10 11

Q: Which one of the following rotations happened?

- $0^\circ$
- CCW $90^\circ$
- CCW $180^\circ$
- CCW $270^\circ$ or CW $90^\circ$

Q: Which data bits were transmitted?
**Example 3**

**Unique Word (UW) for QPSK**

Bit-to-Symbol Map

- $\phi_1$: Q (out of phase)
- $\phi_0$: I (in phase)

Transmitted UW:

\[
\begin{array}{cccccccc}
1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\
+1 & -1 & +1 & +1 & -1 & +1 & -1 & -1 \\
\end{array}
\]

Received UW:

\[
\begin{array}{cccccccc}
1 & 1 & 0 & 1 & 0 & 0 \\
-1 & -1 & +1 & +1 & -1 & -1 & -1 \\
\end{array}
\]

Received Data:

\[
\begin{array}{cccccccc}
1 & 1 & 0 & 1 & 0 & 0 \\
-1 & +1 & +1 & +1 & -1 & -1 & -1 \\
\end{array}
\]

Symbol Index

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</table>

**Q:** Which data bits were transmitted?

**A:** First we need to find out the phase rotation based on the UW.

- Transmitted UW: 10 11 01 00
- Received UW: 11 01 00 10
Example 3 (cont.)

Transmitted UW
10 11 01 00

Received UW
11 01 00 10

Q: Which one of the following rotations happened?

Q: Which data bits were transmitted?
Phase Ambiguity Resolution: Differential Encoding
Commonly used in Satellite Communication

- Instead of inserting 8-bits of UW, we rely on the phase shifts in Diff. Encoding

- Usually, the data are mapped to the phase of the carrier

- Here, the data are mapped to the phase shifts of the carrier signal

- Eq. 7.75, Eq. 7.77-7.79: show that the data symbols $a_0$ and $a_1$ select the phase of the carrier

- Eq. 7.80-7.81: show that the data symbols $a_0$ and $a_1$ select the phase shifts of the carrier
Differential Encoding for BPSK
Differential Encoding: Modulator

Current bit ($n$) defines phase shift (of the carrier signal) from the previous phase based on the encoded bit ($n-1$).
Differential Encoding: Demodulator

Encoded Bit Estimates are produced that must be decoded.
**Example**

- Please follow pages 400-402 of the book

1. **Data Bits-to-Phase Shift Table**
2. **Encoded Bits-to-Symbol Map**
3. **Encoding Rule Truth Table**

**Data Bits: 10110100**

1. **Encoding of Data**
2. **Decoding Rule Truth Table**

**Output when Phase Ambiguity is 0°**

**Output when Phase Ambiguity is 180°**
Differential Encoding for QPSK
Differential Encoding for QPSK

Example

- Please follow pages 402-405 of the book

1) Data Bits-to-Phase Shift Table
2) Encoded Bits-to-Symbol Map
3) Encoding Rule Truth Table

Data Bits: 10 11 01 00 11 10 10 01

1) Encoding of Data
2) Decoding Rule Truth Table

Output when Phase Ambiguity is 0°
Output when Phase Ambiguity is 180°