UNDERGRADUATE RESEARCH STATEMENT

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Although operator algebras and functional analysis, my areas of research, are not generally considered to be especially accessible to undergraduates, we feel that there are three aspects of our current research interests where undergraduate assistance would be helpful.

In 1999, Charles Akemann, Joel Anderson and Nik Weaver came up with the notion of a spectral scale [1]. We describe the setting in the simplest situation. follows. Let M_n be the set of $n \times n$ matrices with complex entries. If $a \in M_n$, we define $\mathrm{Tr}(a)$ to be the sum of the diagonal entries. The adjoint of a, denoted a^*, is the matrix obtained by transposing the rows and columns of a and then taking the complex conjugates of the entries in the transposed matrix. We say that a is self-adjoint if $a = a^*$ and that a is positive if $(az) \cdot z \geq 0$ for every $z \in \mathbb{C}^n$. (Here, $\cdot$ is the scalar product on $\mathbb{C}^n$).

Definition 1. If $b_1, ..., b_k$ are self-adjoint $n \times n$ matrices, then the corresponding **spectral scale** is given by $B(b_1, ..., b_k) := \{(\frac{\mathrm{Tr}(a)}{n}, \frac{\mathrm{Tr}(b_1 a)}{n}, ..., \frac{\mathrm{Tr}(b_k a)}{n}) \mid a \text{ and } 1 - a \text{ are positive matrices } \}$.

In this context, the spectral scale is an attempt to generalize the notion of the spectrum of a matrix to a 'spectrum' of k matrices. The idea can be considerably generalized into an infinite dimensional setting. For further details, see our research statement at http://marcus.whitman.edu/ willsmd/GenResearchInfo/research statement.pdf.

We would like a computer programme that generates graphical representations of spectral scales in the case that $n = 1$ or $n = 2$. This is a situation where undergraduate assistance would be fruitful.

Separately, we have also studied the notion of Hausdorff distance, denoted d_H. The notion of Hausdorff distance provides a way of measuring the distance between two sets in a metric space, X, that takes into account both the shape of the sets, and every point in each set. We discovered that if X is a vector space with a norm, and A and B are bounded, closed, and convex, then

$$d_H(A, B) = d_H(\partial A, \partial B). \tag{1}$$

We would like to find out under what other circumstances equation (1) may hold. For example, are there other metric spaces where we can get similar results for certain types of sets in the space? Some of these investigations would be a good exercise in point-set topology for undergraduate researchers.

References

1. Charles A. Akemann, Joel Anderson, and Nik Weaver, *A geometric spectral theory for n-tuples of self-adjoint operators in finite von Neumann algebras*, J. Funct. Anal. **165** (1999), no. 2, 258–292. MR MR1699015 (2000f:46078)