

Writing a Proof

1. **Theorem.** The sum of any three consecutive integers is divisible by three.

Proof. $x + x + x + x + 2 = 3x + 3 = 3(x + 3)$ □

2. **Theorem.** The sum of any three consecutive integers is divisible by three.

Proof. $x \in \mathbb{Z} \implies x + x + 1 + x + 2 = 3x + 3 = 3(x + 1) \implies$ the sum is divisible by 3. □

3. **Theorem.** The sum of any three consecutive integers is divisible by three.

Proof. Let $x \in \mathbb{Z}$. Then $x + x + 1 + x + 2 = 3x + 3 = 3(x + 1)$. Therefore the sum is divisible by 3. □

4. **Theorem.** The sum of any three consecutive integers is divisible by three.

Proof. Suppose we are given three consecutive integers. We may write them in the form $x, x + 1$, and $x + 2$, where $x \in \mathbb{Z}$. Then

$$x + (x + 1) + (x + 2) = 3x + 3 = 3(x + 1).$$

Since $x + 1 \in \mathbb{Z}$, we see that $x + (x + 1) + (x + 2)$ has the form $3k$, where $k \in \mathbb{Z}$, and thus the sum is a multiple of 3, as desired. □